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Identifying Sorting – In Theory*

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Abstract

Assortative Matching between workers and firms provides evidence of the complementarities or substitutes in production. The presence of complementarities is important for policies that aim to achieve the optimal allocation of resources, for example unemployment insurance. We argue that using wage data alone, it is virtually impossible to identify whether Assortative Matching is positive or negative. Even though we cannot identify the *sign* of the sorting, we can identify the *strength*, i.e., the magnitude of the cross-partial, and the associated welfare loss. We show first that the wage for a given worker is non-monotonic in the type of his employer. This is due to the fact that in a sorting model, wages reflect the opportunity cost of mismatch. We show analytically that this non-monotonicity prevents standard firm fixed effects to correlate with the true type of the firm. We then propose an alternative procedure that measures the strength of sorting in the presence of search frictions. Knowing the strength of sorting facilitates the measurement of the output loss due to mismatch.

Keywords. Sorting. Assortative matching. Complementarities. Supermodularity. Identification.
JEL Codes. J31, C78.

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1 Introduction

Sorting of workers to jobs matters for the efficient production of output in the economy. If there are strong complementarities or substitutes between workers and jobs, the exact allocation has large efficiency implications. In contrast, when complementarities are nearly absent, not much output is lost from randomly allocating workers to jobs. This is important for policy, for example whether we want to design an unemployment insurance program that provides incentives for workers to look for the "right" job instead of accepting the first offer (see for example Acemoglu and Shimer 1999). Complementarities and sorting also have profound implications for wage inequality across differently skilled workers (see for example Sattinger 1975). Sorting based on complementarities is also the driving force in a variety of applications: it is central to the argument of skill-biased technological change; it affects the impact of immigration on the domestic labor force; and it shapes the effect of subsidies to education.

We address two questions in this work. First, we ask whether we can determine if more productive workers are employed in more productive jobs. Second, we ask whether we can determine the magnitude of the loss from mismatch when workers are not employed in the optimal firm. Both questions have attracted recent interest because of the availability of worker-firm match data (see Postel-Vinay and Robin (2006) for an overview) that gives a panel dimension of observations for each worker and firm. When thinking about the answer to each of the questions, we focus on wage data only. While data on profits and output is also available, this is typically reported at the firm level and, therefore, it is not very informative about the productivity of each individual job in the firm.[1]

The first question – whether more productive workers work in more productive jobs – is a positive exercise. It provides insights into the features of the production technology. If the answer is affirmative and there is positive sorting, it means that the inputs in production, worker skill and job productivity, exhibits strong enough complementarities. The marginal product of a worker increases the better the firm is, i.e., the technology is supermodular. Alternatively, if sorting is negative, this provides evidence that inputs in production are substitutes and that the technology is submodular. The canonical example of negative sorting is the work of a consultant, where the better consultants may be needed in the firms that are currently least productive. Unfortunately, from our analysis we conclude that we cannot identify the *sign of sorting*. Based on wage data alone, it is impossible to determine whether sorting is positive or negative. The reason is that wages reflect a worker's marginal product. Under complementarity, this is high in productive jobs while it is low under substitutability. Wages alone thus do not allow us to determine the ranking among firms, and it is not possible to find out whether more able workers derive their higher marginal product from more productive firms or not.

Second, we ask how large the gains are from matching workers to the appropriate firms, and correspondingly, how big the losses are from mismatch. This question about the *strength of sorting* is normative. It allows us to evaluate the efficiency gains from improved job search and other labor market interventions. Our findings provide an affirmative answer: we can identify the strength of sorting. We propose a simple algorithm that allows us to back out (the absolute value of) the degree of complementarity, even without information on the sign of sorting. The main source of identification is the search behavior by workers that differs when the degree of complementarity is high, and when as a

[1] We further discuss the benefits and limitations of additional data beyond wages below.

result, sorting is important.

In addition to answering these two identification questions, we also show that neither of these questions can be answered with the widely cited method that analyzes the correlation between firm and worker fixed effects from wage regressions. This method is first proposed in a seminal paper by Abowd, Kramarz and Margolis (1999), AKM henceforth. The obtained correlation aims to answer the sign and strength questions in one: whether it is positive or negative and how big the coefficient is. The idea is that more productive firms pay higher wages than lower wage firms irrespective of the exact worker they hire, and the firm fixed effect therefore recovers the ranking of the firms. We show analytically that obtaining the firm ranking is not feasible due to the way wages are determined in equilibrium, and as a result, the correlation coefficient is not capable of informing us about the sign nor the strength.

Our analysis builds on the most standard sorting model. We start from the assignment problem analyzed by Koopmans and Beckmann (1957), Shapley and Shubik (1971), and popularized by Becker (1973). In the Beckerian theory, high type workers match with high productivity jobs when there are complementarities. Wages are set competitively and reflect the marginal contribution of a worker's skill. Most importantly, even off the equilibrium allocation, the possible wages paid to mismatched partners are constrained by equilibrium: off-equilibrium wages must be such that they do not induce agents from knowingly deviating from the equilibrium allocation. One innovation of our approach is to explore not only the properties of the wages along the Beckerian allocation, but also of the wages for mismatched agents that happen to "tremble" off the equilibrium path. The properties of those wages by mismatched pairs are crucial for our identification strategy. Once we introduce mismatch as an equilibrium outcome due to search frictions, we can actually link the properties of those wages from mismatch to actually observed wages.

We find that wages of a given worker have an inverted U-shape around the optimal allocation, which corresponds to the frictionless wage. This non-monotonicity reflects the *opportunity cost* of a firm to match with an inappropriate worker type. For a given worker, wages are low if he matches with a "bad" firm, because the value that is generated is low. Maybe less obviously, his wage is also low if he matches with a very "good" firm. The reason is that higher productivity firms have to be compensated for their willingness to match with a "bad" worker because it destroys their opportunity to match with a "good" worker. Under complementarities that firm has a disproportionately larger marginal product with the good worker. This leads to the highest compensation if a worker meets the "right" firm, rather than a wage schedule that is increasing everywhere in the type of firm. To see that the inverted U-shape is actually part of *any* sorting equilibrium with search frictions, observe that the set of eligible partners is bounded by those matches where the match surplus is zero relative to the value of continued search. These bounds in general arise both for low type firms that are too inefficient and for high type firms that rather wait for a better worker, while for intermediate firm types the surplus is strictly positive. Any bargaining procedure that pays wages that are monotonic in the surplus after accounting for the outside option of attracting a more appropriate type will therefore result in wages being non-monotonic in firm type.

Because of the non-monotonic effect of firm type on wages, the wage cannot be decomposed in an additively separable firm and worker fixed effect. We show analytically that the misspecification is not innocuous: for the most common specifications in the literature the firm fixed effect misses any direct connection to the true type of the firm.

In spite of the fact that we cannot identify the *sign* with any procedure, we develop a method that enables us to identify the *strength* of sorting. We thus offer an alternative to remedy the shortcomings of the fixed effects regression. Identification derives from the distinct features of the search behavior of workers under different degrees of complementarities. First, we extract from the range of wages paid what the cost of search is. The highest observed wage corresponds to the wage obtained in a frictionless market and we use this to order the workers and obtain the type distribution. Likewise, we can obtain an order of the firms by the level of wages that they pay. The difference between the highest and the lowest wage corresponds to the cost of search. Second, given the search cost, the fraction of the firm population that an agent is willing to match with, i.e., the matching set, identifies the strength of the complementarity as expressed by the (absolute value of the) cross-partial of the production function. This is possible because the strength of the cross-partial directly reflects the output loss due to mismatch. We can relate our method of identification to the analysis in Gautier and Teulings (2004, 2006) of second-order approximations to infinite-horizon search models. We show that identification is ensured without knowledge of the sorting pattern, without approximations to the case of negligible search costs, and in the presence of type-dependent costs, and we can do this in few transparent steps. But this comes at the cost of simplified economic and econometric modeling assumptions.

The setup of our economy is very simple with dynamics reduced to two periods. The objective is to solve analytically what the standard infinite horizon models cannot deliver. This makes the models very specific, but in section 5 we show that our insights extend to the fully dynamic steady state models. We also discuss alternative models and argue that whenever wages reflect the competitively determined outside option, they must necessarily be non-monotonic in firm type.

Related Literature. Given the importance of sorting, a large body of recent empirical literature has estimated whether sorting is positive or negative. As mentioned above, this renewed interest has been catalyzed by the availability of worker-firm match data, surveyed e.g., in Postel-Vinay and Robin (2006). Using the AKM method, several papers find an insignificant or even negative correlation in fixed effects between worker and firm types. This result has been replicated for a number of countries including France, US, Denmark and Brazil. The result is taken as indication that Positive Assortative Matching between workers and firms does not play a major role in the labor market.

AKM use matched employer-employee data to decompose wages into different effects related to worker and firm characteristics. With unrestricted correlation among the effects they are able to estimate the firm and worker components of wages. The generality of their econometric approach allows for many different estimation techniques to determine each of the components. A by-product of their contribution is a simple empirical measure of sorting that can be obtained by estimating a log-wage equation in which wages are a function of a worker fixed effect, a firm fixed effect, and an orthogonal error term: $\log w_{it} = a_{it}\beta + \delta_i + \psi_{j(i,t)} + \varepsilon_{it}$, where w_{it} denotes the wage, a_{it} are time varying observables of workers, δ_i is a worker fixed effect, ψ_j is the fixed effect of the firm j at which worker i is employed at time t, and ε_{it} is an orthogonal residual. That is, ψ_j captures the average effect that a firm has on the wages of the workers that are willing to match with it. The correlation between δ_i and ψ_j in a given match is taken as an estimate of the degree of sorting.

We show in our theoretical exercise that the assumption that the firm effect is independent of the worker's type is theoretically not justified in this setting. In particular, for those workers who are matched with a firm that has a lower rank than their own, the wage increases when the firm type increases because the worker-firm "fit" improves. In contrast, workers who are matched with a higher ranked firm see a decrease in the wage when the firm becomes better because the worker-firm "fit" deteriorates. We argue above that this implies that the correlation between the firm and worker fixed effect cannot be taken as a measure of sorting.

The AKM findings have been extensively cited in the literature. In the first instance, the methodology has been widely used to analyze sorting for many different countries. It has repeatedly been established in many matched employer-employee data that there is a small, or insignificant correlation between the worker and firm fixed effects. The methodology has also been extended to include different aspects of sorting. For example, Combes, Duranton, Gobillon (2008) use the AKM methodology to estimate the geographical degree of sorting.

In addition, there is an extensive literature leaning on the AKM (1999) fixed effects regression, both in empirical and theoretical work. The fact that the correlation between the worker and firm fixed effect is small is interpreted in different ways. For example in empirical work, Postel-Vinay and Robin (2002) analyze wage heterogeneity in a search model. They have no complementarities within occupations and cite AKM as evidence for the absence of sorting. In contrast, Bender and von Wachter (2006) argue that the small but significant correlation coefficient obtained for the French labor market is indeed evidence in favor of sorting. Similarly, theoretical work citing the results of AKM interpret them either way. Anderson and Smith (2010) for example justify the absence of sorting to motivate a matching model with learning that does not generate sorting in equilibrium. They cite AKM as evidence. Shimer (2005) justifies the use of complementarities and the resulting sorting based on AKM despite them finding a small effect. The interpretation is that the workers' unobserved characteristics have a small but statistically significant effect on firm profits, i.e., there is sorting. Cabrales, Calvó-Armengol and Pavoni (2008) also cite AKM to argue that the degree of sorting is on the increase over time.

Beyond the use and citation of the AKM method and results, other work has singled out shortcomings of the fixed effects regression. Simulations of search models with strong complementarities and sorting nonetheless generate small or even negative correlations of the simulated fixed effects of workers and firms. Lopes de Melo (2008), Lise, Meghir and Robin (2008), and Bagger and Lentz (2008) study variations of structural labor search models with an infinite horizon, and simulate as well as estimate those models with matched employer-employee data. Our objective is to provide a much simpler framework, but one that allows us to investigate theoretically how to measure the extent of sorting and to derive the correlation between fixed effects analytically. Lopes de Melo (2008) also reports correlations between worker fixed effects as a measure of sorting, and our approach highlights why this captures some (but not all) the relevant information on the importance of sorting. In particular, it captures the range in which workers accept jobs, since a narrow acceptance range means that the workers at a firm are rather similar. This arises when sorting is very important, but may also arise simply because search costs are small, and a careful consideration of both forces is necessary.

And in an approach related to our own, Gautier and Teulings (2004, 2006) allow for an infinite horizon and a richer econometric environment with measurement error while relying on second-order approximations. They restrict the match surplus function to insure that matching is positive assortative, and thus cannot address whether their framework permits the identification of the sign or the strength if that restriction were relaxed. Given our results, we suspect that recovering the sign will be infeasible even in their model.[2] On the other hand their estimates about the strength of sorting will remain valid even if one is agnostic about the matching patterns, as long as search costs are type-independent (below, we develop alternatives for type-dependent search costs within our framework). Thus, we believe that there is a multitude of ways (such as our approach, Gautier and Teuling's second-order approximations, or other approaches under sign restrictions) that allow progress on the main policy-relevant issue without relying on the exact knowledge of the sorting pattern (for which additional data might be required). We return to this in the discussion section.

One alternative approach is to use profit data in addition to wage data. Clearly, if we have information on prices from both sides of the market, we know the total output for all matched pairs and therefore the technology. This immediately allows us to back out both the sign and the strength of sorting. This approach has been taken by Haltiwanger, Lane and Speltzer (1999), van den Berg and van Vuuren (2003), Mendes, van den Berg, Lindeboom (2007) to estimate the performance of search models. With sorting, this immediately leads to identification. The problem is that in practice this is difficult because output and profit measures are usually only provided at the level of the firm or the establishment, and attributing them to each individual worker is difficult. In the absence of job level profit data, one would need a theory of the firm with heterogenous agents (for example Eeckhout and Pinheiro (2009)) to attribute the firm level profits to each individual job. Instead, the objective of our paper is to investigate the extent to which the two questions above can be answered from wage data alone. This has the great advantage that data on wages is available for each individual worker and is of high quality.

Finally, there is also an extensive literature on hedonic models that are thought to be not identified. Hedonic models characterize the pricing of goods that consist of bundles of attributes (housing, for example). The focus is on multidimensional characteristics and the patterns of sorting based on those characteristics. Recently, Ekeland, Heckman and Nesheim (2002, 2004) show that the hedonic model is generically nonparametrically identified, despite the fact that the sorting equilibrium in a single market implies no exclusion restrictions. They show that the commonly used linearization strategies cause the identification problem because the hedonic model is generically nonlinear. In this class of models with multidimensional characteristics, Dupuy (2010) observes that in a combined matching-hedonic model with sorting on both skills and preferences, wages are a function of both preferences and worker attributes. This can result in non-positive dependence of wages on firm types since the firm types do not constitute a complete order common to all agents. Of course, one can make this observation even in the simple matching model by relaxing the monotonicity assumption, i.e., output is not everywhere increasing in firm and/or worker type (see also section 5).

[2] We discuss the case of discounting explicitly in Section 5. Allowing for discounting rather than fixed search costs will open some avenue for sign-identification, but this possibility vanishes with a decreasing discount rate between matching opportunities, which is assumed in most approximation-strategies.

2 The Model

The basic primitives of our simple matching model build on Becker (1973). There is a unit mass of workers and a unit mass of firms. Workers and firms are heterogeneous in terms of their productivity. Workers draw their type x from distribution $\Gamma(x)$ with smooth density $\gamma(x)$ on $[0,1]$. Firms draw their type y from distribution $\Upsilon(y)$ with smooth density $\upsilon(y)$ on $[0,1]$.

When types x and y form a match, they produce positive output $f(x,y) \geq 0$ whilst having an outside option of remaining unmatched. We assume that workers and firms can be ranked in terms of their productivity, i.e., $f_x > 0$ and $f_y > 0$. Then it is without loss of generality to index a worker by his *rank* in terms of productivity, i.e., by the fraction of workers that are less productive then him. Similarly, we can identify each firm by its rank in the distribution of firm productivities. This means that $\Gamma(\cdot) = \Upsilon(\cdot) = x$, i.e., the distributions are uniform. Assume that workers who do not get matched obtain a payoff of zero, and since output is non-negative, all agents will prefer to match.

For the assignment of workers to firms the cross-partial of the production function is important. We do not restrict the sign of the cross-partial since this will be instrumental in determining whether there is positive or negative assortative matching. Denote by $\mathcal{F}$ the class of all functions f that are monotonic: $f_x, f_y > 0$; and that have a monotonic marginal product: $f_{xy}(x,y)$ is either always positive or always negative.[3] The assumption that the cross-partial does not change sign allows us to unambiguously talk about positive or negative sorting. Production functions with complementarities ($f_{xy} > 0$) are in set $\mathcal{F}^+ \subset \mathcal{F}$. Production functions with substitutes ($f_{xy} < 0$) are in set $\mathcal{F}^- \subset \mathcal{F}$.

To illustrate the implications of our analysis we will illustrate our results for the following examples of production functions

$$f^+(x,y) = \alpha x^\theta y^\theta + h(x) + g(y), \quad (1)$$
$$f^-(x,y) = \alpha x^\theta (1-y)^\theta + h(x) + g(y), \quad (2)$$

where $g(\cdot)$ and $h(\cdot)$ are increasing functions and $\alpha \geq 0$ and $\theta > 0$ are parameters that indicate the strength of the complementarities. We assume that $g(y)$ is such that higher type firms produce higher output even under the second specification. It is obvious that $f^+ \in \mathcal{F}^+$ and $f^- \in \mathcal{F}^-$.

2.1 The Frictionless Environment

In the absence of frictions the matching market is competitive. An assignment of workers x to firms y is denoted by μ, i.e., $\mu(x) = y$ means that worker x gets hired by firm y. A market equilibrium specifies an assignment μ between x's and y's and some wage schedule $w(x,y)$ that determines the split of output between the worker and the firm that are matched. The payoff to the worker is $w(x,y)$ and the payoff to the firm is $\pi(x,y) = f(x,y) - w(x,y)$. Both workers and firms take the wage schedule as given. The tuple of functions (μ, w) is an equilibrium if there is no worker-firm pair that could do better by matching amongst themselves than with their current partners,[4] i.e.,

$$w(x,\mu(x)) + \pi(\mu^{-1}(y), y) \geq f(x,y), \quad \forall x, y \quad (3)$$

[3] Later, in section 5 we discuss the virtues of relaxing these assumptions.

[4] It is well-known that a strict cross-partial yields a one-to-one mapping $\mu(\cdot)$ in equilibrium. In general $\mu(\cdot)$ is a correspondence, with the equilibrium definition extended to all pairs in that correspondence.

and no agent prefers to remain single, i.e., $w(x,\mu(x)) \geq 0$ for all x and $\pi(\mu^{-1}(y),y) \geq 0$ for all y.

We derive the main prediction of Becker's (1973) model concerning the wages in the economy. Rearranging (3) such that only profits are on the left hand side and recalling that $\pi(x,y) = f(x,y) - w(x,y)$ immediately reveals the equilibrium profits for firm y must satisfy:

$$\max_x f(x,y) - w(x,\mu(x)).$$

This yields the first order condition

$$f_x(x,y) - \frac{dw(x,\mu(x))}{dx} = 0. \tag{4}$$

In equilibrium this has to hold evaluated at $y = \mu(x)$, and therefore the equilibrium wage scedule $w^\star(x) := w(x,\mu(x))$ can be obtained by integrating (4) along the equilibrium path:

$$w^\star(x) = \int_0^x f_x(\tilde{x},\mu(\tilde{x}))d\tilde{x} + w_0, \tag{5}$$

where the constant of integration $w_0 \in [0, \min f(x,y)]$ can be thought of as some exogenous bargaining rule that splits the surplus between the lowest types in case these types have a positive surplus over remaining single.[5] Observe that the worker obtains exactly his marginal product along the equilibrium allocation. Therefore, equilibrium profits of type y are given by output minus the wage $w^\star$ with the optimal worker $\mu^{-1}(y)$. This can be re-written as

$$\pi^\star(y) = \int_0^y f_y(\mu^{-1}(\tilde{y}),\tilde{y})d\tilde{y} + f(0,0) - w_0. \tag{6}$$

Furthermore, we know from Becker's analysis that matching is positive assortative when the production function is supermodular ($f_{xy} > 0$), in which case $\mu(x) = x$. Under submodularity ($f_{xy} < 0$) in equilibrium the matching is negative assortative and $\mu(x) = 1-x$, since lower type firms have a higher *marginal* value for better workers and are willing to pay more for them.

2.2 On the equilibrium path

We show that in this simple competitive model the sign of sorting – i.e., the sign of the cross-partial – cannot be identified from wage data alone. We will first illustrate the result by considering our restricted class of production functions outlined above and then present the general theorem. Suppose the underlying production technology is not known and the true technology is either one of the two example technologies f^+ given in (1) or f^- given in (2). By (5) the wages under f^+ and f^- are

$$\begin{aligned} w^{\star,+}(x) &= \int_0^x f_x^+(\tilde{x},\tilde{x})d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0 \\ w^{\star,-}(x) &= \int_0^x f_x^-(\tilde{x},1-\tilde{x})d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0. \end{aligned}$$

[5] When $f(0,0) = 0$, then $w_0 = 0$ and the wage schedule is uniquely determined. Otherwise there are a continuum of competitive equilibria associated with different w_0, and we assume that the specific split w_0 is a primitive determined by some exogenous bargaining rule.

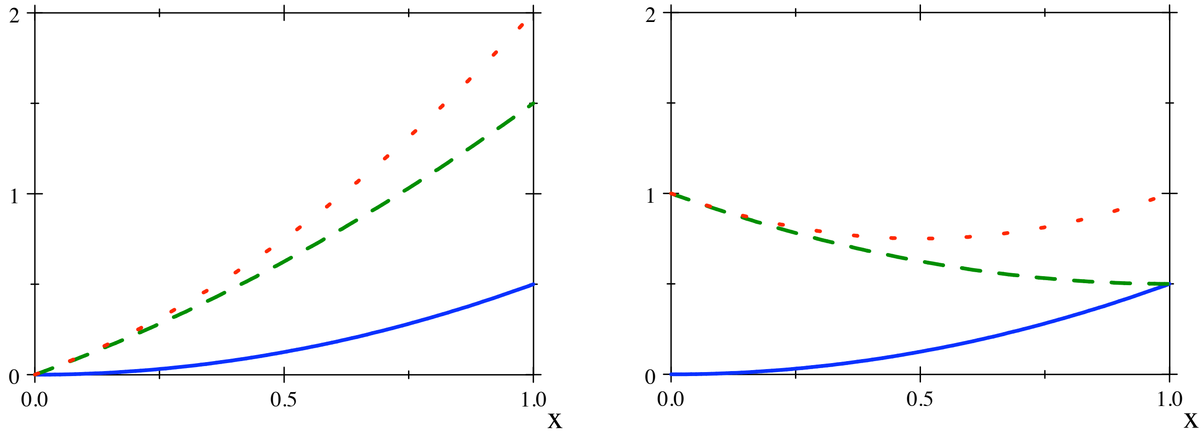

Figure 1: The solid line gives equilibrium wages $w(x, \mu(x))$, the dashed line profits $\pi(x, \mu(x))$, and the dotted line total output $f(x, \mu(x))$ under $f^+ = xy + y$ [left] and $f^- = x(1-y) + y$ [right] with $w_0 = 0$.

Under both technologies the wages on the equilibrium path are exactly identical, and from wage data alone one cannot distinguish between positive and negative sorting. The problem is obtaining the order of the firms. If we only have wage data and no profit data, and we derive the order on the firms by ranking them by increasing wages, we will obtain two different orders depending on whether we have complements or substitutes. To see this, observe that under positive assortative matching (henceforth PAM) higher type firms pay higher wages along the equilibrium path whereas under negative assortative matching (NAM) higher type firms pay lower wages. In the former $w(y, y) = \frac{\alpha y^{2\theta}}{2}$ is increasing in y, in the latter $w(1-y, y) = \frac{\alpha(1-y)^{2\theta}}{2}$ is decreasing in y. This result is true for any general production technology as summarized in the proposition that follows below.

In Figure 1 the solid line shows for each worker x his wage, the dashed line shows the profits of the firm that is matched to worker x, and the dotted line gives the total output that they produce. The left panel depicts these for the production function $f^+ = xy + y$, while the right panel covers the outcomes for production function $f^- = x(1-y) + y$. If one only observes the wages according to the solid line, both cases look identical. The cases only differ in profits and total outputs. For example, the dashed line of the profits of the firm matched to worker x is decreasing in worker type x only under f^-. While higher y firms have higher profits, in this case higher x workers are matched with lower y firms who obtain lower profits. That is, even under NAM $\pi^{\star,-}(y) = y + \frac{(1-y)^2}{2}$ is increasing in y even though $\pi^-(x, \mu(x))$ is decreasing in x. The payoffs under both technologies are summarized in the following table.

	$f^+ = xy + y$	$f^- = x(1-y) + y$
$w(x, \mu(x))$	$\frac{x^2}{2}$	$\frac{x^2}{2}$
$\pi(x, \mu(x))$	$\frac{x^2}{2} + x$	$\frac{x^2}{2} + 1 - x$
$f(x, \mu(x))$	$x^2 + x$	$x^2 + 1 - x$

Proposition 1 *For any production function $f \in \mathcal{F}^+$ that induces positive sorting there exists a production function $f \in \mathcal{F}^-$ that induces negative sorting and the equilibrium wages $w^*(x)$ are identical under both production functions.*

Proof. In Appendix. ■

2.3 Off the equilibrium path

Identification needs variation. Identification of sorting from equilibrium wages may be difficult simply because there is no independent variation across firms and workers. In the frictionless case workers sort perfectly in the sense that each type of firm attracts exactly one worker type. Even if workers became unemployed and could match again later without frictions, the panel dimension would not allow us to identify a separate effect for firms and workers, because workers will always end up in the same type of firm. There would not be any wage variation, and it cannot be identified whether a high wage is due to the worker ability or the firm productivity.

Here we entertain the idea that workers "tremble" to off-the-equilibrium firms. Without specifying how those wages are determined, we start from the premise that such off-equilibrium wages are observed occasionally. This gives additional variation that one might suspect to be crucial for identification. This simple exposition is also useful to build intuition for the results in more realistic environments because it has a close connection to the mismatch that occurs under search frictions, as will become clear in the next section.

And even though those wages obtained when "trembling" are not equilibrium wages, they must not induce agents on the equilibrium allocation to deviate. Therefore, those wages $w(x, y)$ must satisfy the on-the-equilibrium path restriction (3) as before: $f(x, y) = w(x, y) + \pi(x, y) \leq w(x, \mu) + \pi(\mu^{-1}, y)$. A condition with more bite is obtained by requiring for a given wage schedule $w(x, y)$ that there should be no *individual* deviation by either worker or firm type:

$$f(x, y) - w(x, y) \leq \pi(\mu^{-1}(y), y) \tag{7}$$

$$w(x, y) \leq w(x, \mu(x)). \tag{8}$$

This is the standard competitive equilibrium notion of a matching market, in which each agent takes the price (here: wage) schedule as given and assumes that he can obtain a partner at this transfer price. For markets to clear the price has to be such that no agent individually wants to choose a different partner.

For a given (x, y) combination, call the set of wages that are consistent with (7) and (8) $W(x, y)$. This wage schedule is not uniquely determined and wages range between the lowest wage that is just high enough to prevent firms from deviating and the highest wage that is just low enough to prevent workers from deviating.

It is important to note the implication of (8): wages are highest at the firm that is most appropriate for the worker. Even under positive assortative matching, a worker x who tries to trade with a firm that is higher (or lower) than his optimal type $\mu(x)$ will earn lower wages. At less productive firms this arises for the obvious reason that the surplus is too low. At more productive firms this arises because the firm forgoes the benefit of hiring the more appropriate worker and has to be compensated for this opportunity cost.

Even with observation of wages from mismatched pairs, we cannot determine the sign of sorting. It is easily verified that in the case of the technology $f^+(x,y)$ any wage $w(x,y)$ in $W(x,y)$ satisfies

$$\alpha\,(xy)^{\theta} - \frac{\alpha}{2}y^{2\theta} \leq w(x,y) - h(x) \leq \frac{\alpha}{2}x^{2\theta}. \tag{9}$$

In the case of $f^-(x,y)$ any wage $w(x,y)$ in $W(x,y)$ satisfies

$$\alpha x^{\theta}(1-y)^{\theta} - \frac{\alpha}{2}(1-y)^{2\theta} \leq w(x,y) - h(x) \leq \frac{\alpha}{2}x^{2\theta}, \tag{10}$$

which is identical to (9) if we misinterpret the types as $\hat{y} = 1 - y$. If we have no information on profits, as before we cannot derive the order on y simply from wage data, not even after observing off-the-equilibrium path wages. The bounds on the wages under PAM and NAM are identical if we use the order on wages to derive the order on y (in which case under NAM we assign the order $1-y$ to the firms). The static Beckerian model will therefore not allow for identification of assortative matching based on wage data alone.

Proposition 2 *For any production function $f \in \mathcal{F}^+$ that induces positive sorting there exists a production function $f \in \mathcal{F}^-$ that induces negative sorting and the equilibrium wage sets $W(x,y)$ in the former are identical to equilibrium wage set $W(x,1-y)$ in the latter.*

Proof. The proof follows the same argument as in Proposition 1. ■

3 Mismatch due to Search Frictions

We now consider an extended model with mismatch due to frictions caused by delay. Search frictions give a structural reason why workers may match with firms even if these firms would not be exactly optimal in the absence of frictions. Unlike the static Beckerian model, search frictions may induce different behavior in the acceptance decision of matches. First, we derive the equilibrium allocation in the presence of search costs. We relate this to the previous section and find that there is a strong resemblance, even though the search frictions give an explicit bound on the mismatch that prevents very inefficient matches to materialize. We address the issue of identification of positive/negative sorting in this model, and whether we can identify the sign of sorting from wage data alone. Second, for this model we analytically derive the firm fixed-effect. We postpone the discussion of identification of the strength of sorting to the next section.

3.1 Introducing Search Costs

Our model has hiring in two stages. In stage one, each worker is paired with one firm. The pairings are random. One can think of this part of the hiring process as standing in for some connections that workers have to the labor market prior to engaging in an extensive search for jobs. The pair can either agree to stay together at some wage, or search for a better partner. Firms have fixed types and cannot exit and reenter to take another draw. Those pairs who decide not to stay together each incur a search cost c due to the delay. The formulation with a constant search cost follows Chade (2001) and Atakan (2006). In the second stage, all remaining agents are matched according to the competitive, frictionless allocation as outlined above.[6] After the search process has ended production starts. To allow for a panel dimension in the observations (i.e., over time, each worker matches with more than one firm, and each firm matches with more than one worker) we assume that each agent goes through this two-stages hiring process several times in his life.

We consider the same class of production functions $\mathcal{F}$. For exposition it will be convenient to restrict the supermodular function in $\mathcal{F}^+$ to functions with symmetric cross-partial such that $f_{xy}(x,y) = f_{xy}(y,x)$. For submodular functions in $\mathcal{F}^-$ which induce negative sorting it will be convenient to restrict attention to symmetry of the form $f_{xy}(x,y) = f_{xy}(1-y, 1-x)$.

We assume that the transfer in the first period is determined by Nash bargaining with equal bargaining weights. We illustrate this with our example production function f^+ in (1). When a worker x meets a firm y, the payoff from matching is $f(x,y)$. Waiting until next period and matching in the perfectly competitive labor market yields payoff $w(x,\mu(x)) - c$ to the worker and $\pi(\mu^{-1}(y), y) - c$ to the firm. A first-period match will therefore be accepted provided that the current match surplus over waiting is positive. This gives the following bounds on the degree of mismatch where a match is still tolerated:[7]

$$f(x,y) - (w^\star(x) + \pi^\star(y) - 2c) \geq 0. \tag{11}$$

For a given firm y we call the set of worker types that fulfill (11) his acceptance set and denote it by $A(y)$ for the firms and by $B(x)$ for the workers.[8] Similar to the work by Atakan (2006) we can show that the bounds of this set are increasing if the production function is in $\mathcal{F}^+$ and decreasing if it is in $\mathcal{F}^-$, which naturally extends the notion of sorting to sets.

For our example production function f^+ in (1) it is easy to verify that (11) reduces to

$$\alpha(xy)^\theta - \frac{\alpha}{2}x^{2\theta} - \frac{\alpha}{2}y^{2\theta} \geq -2c. \tag{12}$$

and therefore the acceptance set becomes $A(y) = \left[(y^\theta - 2\sqrt{c/\alpha})^{1/\theta}, (y^\theta + 2\sqrt{c/\alpha})^{1/\theta}\right]$. Due to symmetry the acceptance set of the workers looks identical. The range of mutually accepted matches is illustrated by the area between the dashed lines in Figure 2 for the case $\theta = 1$.

[6] Here we need to worry about the possibility that when the acceptance sets span the entire type space (e.g., because of high search costs or low complementarities), no agents are left in the second stage, in which case the continuation payoff is not determined. Without modeling this explicitly, we think of a tremble that ensures that there are always some agents who end up in the next period. For many parameters each worker and firm type rejects some agents on the other side of the market, and these agents will indeed move to the second stage.

[7] It may well be that for low types the surplus in the next period does not exceed the total waiting cost of $2c$. In order to avoid keeping track of endogenous entry, we assume that people will search even if that is the case. This may be due to the fact that the outside option (e.g. unemployment benefits) are contingent on searching. This issue never arises when $f(0,0) - w_0 > c$ and $w_0 > c$, as all agents than have an incentive to search, or when search costs are proportional as in Section 5.

[8] Note that for supermodular functions in $\mathcal{F}^+$ this acceptance set is identical for workers and firms of the same type, which is a general consequence of the symmetry of f_{xy}. Therefore the distribution of types in the second stage is identical for workers and firms, and therefore the equilibrium assignment is still $\mu(x) = x$. Similarly, for submodular functions in F^- is can be shown that under our symmetry condition when x accepts y then $\hat{y} = 1 - x$ accepts $\hat{x} = 1 - y$, which leads to distributions that are symmetric around $1/2$ and the equilibrium assignment indeed remains $\mu(x) = 1 - x$.

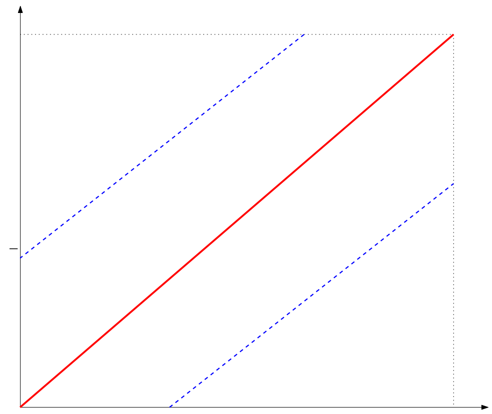

Figure 2: Acceptance sets with type-independent search costs for $\theta = 1$.

Because the surplus is divided equally, the worker obtains half of this surplus on top of his outside option that is given by his value of waiting. Therefore, his wage is

$$\begin{aligned} w(x,y) &= \frac{1}{2}\left[f(x,y) - w(x,\mu(x)) - \pi(\mu^{-1}(y),y) + 2c\right] + w(x,\mu(x)) - c \\ &= \frac{1}{2}\left[f(x,y) + w(x,\mu(x)) - \pi(\mu^{-1}(y),y)\right]. \end{aligned} \tag{13}$$

It is straightforward to see that this wage is exactly in the middle of the acceptance set $W(x,y)$ outlined in (7) and (8) for off-equilibrium-wages of the frictionless model. It immediately implies that positive and negative sorting cannot be identified by observed wage data, because the wages under supermodular production functions coincide with those under submodular production functions (under misinterpretation of the unobserved firm type).

Proposition 3 *For every supermodular production function $f^+ \in \mathcal{F}^+$ that induces positive sorting there is a submodular production function $f^- \in \mathcal{F}^-$ that induces exactly the same wages for workers when we reinterpret firm types as $\hat{y} = 1 - y$.*

Proof. Wages in the second period coincide by Proposition 1. Wages in the first period coincide because they are in the exact arithmetic middle of the wage set $W(\cdot,\cdot)$ which by Proposition (2) coincide under the re-interpretation. ■

For our example production technology f^+ we get as wages

$$w(x,y) = \frac{\alpha}{2}(xy)^\theta + \frac{\alpha}{4}x^{2\theta} - \frac{\alpha}{4}y^{2\theta} + h(x).$$

For some of the results it will be instructive to rewrite the wages as a function of the distance k between the worker and the firm, which for the special case of $\theta = 1$ becomes particularly tractable:

$$w(x, x-k) = w(x, x+k) = \frac{\alpha}{2}x^2 - \frac{\alpha}{4}k^2 + h(x).$$

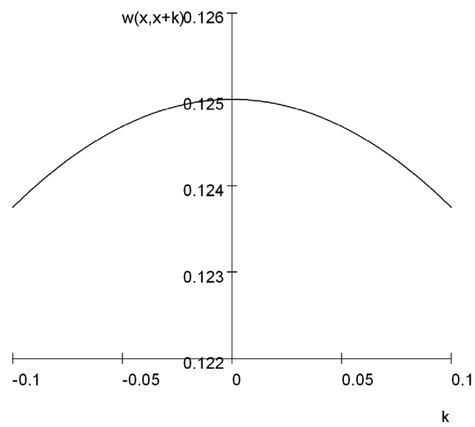

Figure 3: First period wages under mismatch with a type $y = x + k$ that is k away from the optimal match. [Parameter values $x = .5,\ c = .25,\ \alpha = \theta = 1,\ h(x) = 0.$]

Now it becomes immediately apparent what we observed above: there exists an ideal firm in the interior of the set of matches where wages are highest. In this example, a worker has the highest wage when matching with a firm with identical type ($k = 0$), and loses quadratically with the distance to the firm. The reason is that a worker who matches with a firm that has too low a type does not produce a lot of output. On the other hand, a worker who wants to induce a much better firm to match with him has to compensate the firm for not matching with a more appropriate worker. Therefore a worker is not necessarily better off matching with a higher type firm. In a large region – i.e., whenever the firm is higher ranked than the worker – wages fall by matching with even better firms. Figure 3 illustrates the wage schedule of a worker as a function of the distance to the firm he matches with, and highlights the fact that the wage falls in firm type in part of the region. This result holds more generally:

Proposition 4 *For each $x \in (0, 1)$ wages $w(x, y)$ are non-monotone in y.*

Proof. Wages $w(x, y)$ are in the exact arithmetic middle of the wage set $W(x, y)$. Since worker type x chooses the optimal wage, by (8) any wage in $W(x, y')$ is lower than the wage $w(x, \mu(x))$ under the optimal assignment. Since $W(x, \mu(x)) = w(x, \mu(x))$ the optimal wages arise in the first stage. Because search costs are positive, all firm types close to $\mu(x)$ have a positive surplus and thus will form a match with x in the first stage. Therefore, wages are non-monotone around the optimum wage $w(x, \mu(x))$. ■

Note that none of the results depend on an equal split of the surplus or identical search costs for workers and firms. We consider the case of asymmetric bargaining shares and different, type-dependent search costs later. In what follows, we show that this non-monotonicity of the wage schedule makes it impossible for a fixed effect estimator to detect the sign and the strength of sorting.

3.2 Inconclusive Firm-Fixed-Effects

In this section we assess the ability of the fixed effects approach that we discussed in the introduction to detect the strength of sorting, i.e., the magnitude of the cross-partial. We already know that due to the non-monotonicity of equilibrium wages the correlation between worker and firm fixed effects will provide a misspecified measure of the degree of complementarity. Nonetheless, one might conjecture that some misspecification is unavoidable and the approach might nevertheless pick up at least the right qualitative properties. In this section we show for the most popular example in the literature that the fixed effect of the firm does not correlate at all with its true underlying type. Therefore, the correlation between worker and firm fixed effects will also be zero and uninformative about either the sign or strength of sorting. This result in particular involves verifying how the boundaries of the acceptance sets change in types, in addition to the change in the wage profile.

We will consider the fixed effect approach assuming a long panel of observations for each worker and firm. That is, each worker and firm goes through our two step process many times. The fixed effects approach decomposes the wage when worker x matches with firm y into the sum of the worker's fixed effects $\delta(x)$ and the firm's fixed effect $\psi(y)$ plus a residual ε_{xy} :[9]

$$w(x,y) = \delta(x) + \psi(y) + \varepsilon_{xy}, \tag{14}$$

For this approach, we require the fixed effects to be unbiased. We show in the appendix that this is the case when

$$\delta(x) = \int_{B(x)} \left[w(x,y) - \psi(y)\right] d\Upsilon(y|x), \tag{15}$$

$$\psi(y) = \int_{A(y)} \left[w(x,y) - \delta(x)\right] d\Gamma(x|y), \tag{16}$$

where $A(y)$ and $B(x)$ are the acceptance sets, and $\Upsilon(y|x)$ and $\Gamma(x|y)$ are the distributions conditional on being in the acceptance set (with densities $\upsilon(y|x)$ and $\gamma(x|y)$).[10] Substituting (15) into (16) we can write the firm fixed effect as

$$\psi(y) = \underbrace{\int_{A(y)} \left[w(x,y) - w_{av}(x)\right] d\Gamma(x|y)}_{=:\Psi(y)} + \int_{A(y)} \int_{B(x)} \psi(\tilde{y}) d\Upsilon(\tilde{y}|x) d\Gamma(x|y) \tag{17}$$

where $w_{av}(x)$ is the average wage of worker x, defined by $w_{av}(x) = \int_{B(x)} w(x,y) d\Upsilon(y|x)$. The fixed effect is therefore determined by a differential equation. This equation is governed by the characteristic term $\Psi(y)$ that captures the intuitive aspect that the firm fixed effect is the difference between the wage that the firm pays and the average wage that the worker receives. If the characteristic term $\Psi(y)$ is constant, it is immediate that the solution to (17) yields a constant fixed effect $\psi(y)$ that does not vary across firms.[11] That is, in this case the fixed effect does not pick up the differential willingness to pay by firms to hire better workers.

[9] Our analysis is in levels rather than in log-wages. The same decompositions (15)–(18) obtain similarly if we replaced the wage by the log-wage and the average wage by the average of the log-wages, and the qualitative results that the characteristic part of the fixed effect can be increasing, zero, or decreasing depending on the exact specification of the type distribution arises in a similar way. Only the exact shape of the type distribution is more complicated.

[10] Let $\underline{a}(y) = \inf A(y)$ and $\bar{a}(y) = \sup A(y)$, then $\Gamma(x|y) = \Gamma(x)/[\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y))]$. Similar for $\Upsilon(y|x)$.

[11] The characteristic term $\Psi(y)$ is constant iff it is zero because the integral over the wages minus the average wage is zero. Then (17) is solved trivially when $\psi(y)$ equals zero.

To show that fixed effects are not suited to analyze sorting in this model, we consider a version of our example production function: $f(x,y) = \alpha xy + h(x) + g(y)$, where $\alpha > 0$. This yields acceptance sets of form $A(y) = [y-K, y+K]$ with $K = 2\sqrt{c/\alpha}$. These acceptance sets are independent of the type distributions Γ and Υ. To preserve the simple structure of the acceptance set for illustration purposes, it will be convenient to allow the type distribution to possibly be non-uniform at this point rather than to normalize them and to change the production function accordingly.

For this example we will calculate how the characteristic part of the fixed effect $\Psi(y)$ changes in y. For the formal analysis of the characteristic part we focus on firms $y \in (2K, 1-2K)$, because their acceptance set as well as the acceptance sets of the workers they match with is in the interior of the type space. That will avoid cumbersome discussions of corner properties, which can be shown to have negligible impact if c is small. In the interior part we obtain

$$\begin{aligned}\Psi'(y) \quad = \quad & \int_{y-K}^{y+K} \frac{\partial w(x,y)}{\partial y} \gamma(x|y) dx \\ & + (w(y+K,y) - w_{av}(y+K))\, \gamma(y+K|y) \\ & - (w(y-K,y) - w_{av}(y-K))\, \gamma(y-K|y).\end{aligned} \tag{18}$$

Call the first term $\Psi_1'(y)$ and the second and third term $\Psi_2'(y)$ so that $\Psi'(y) = \Psi_1'(y) + \Psi_2'(y)$. The first term $\Psi_1'(y)$ accounts for the wage change across those workers with which this firm type matches. The second and third term $\Psi_2'(y)$ reflect that the matching set is slightly changing when the firm type changes.

We obtain the following results when the type distribution has a linear density:[12]

$$\Psi_1'(y) \quad > \quad 0 \ \text{ if } \ \gamma'(x|y) > 0 \ \text{ for all } \ y; \tag{19}$$

$$\Psi_2'(y) \quad < \quad 0 \ \text{ if } \ \gamma'(x|y) > 0 \ \text{ for all } \ y. \tag{20}$$

The first effect arises because the wage change induced by a higher firm type is non-monotonic: $\frac{\partial w(x,y)}{\partial y} = \frac{\alpha}{2}x - \frac{\alpha}{2}y$, decreasing when $x < y$ and increasing when $x > y$. In this case it depends whether there are more high or low worker types to determine which effect dominates, and for uniform type distributions this effect is exactly zero.[13] In general this first effect is counteracted by the second effect that arises because the boundaries of the acceptance set change. Low types leave the acceptance set and high types join it. At the boundaries workers earn the lowest wages, and therefore an increasing density means that the firm gains more low wage workers than it loses and its fixed effect tends to decrease, while for opposite density it loses more low types than it gains high types and its fixed effect increases. We show this formally in the Appendix. Again this effect is zero if the type distribution is uniform.

[12] Linear densities imply that the conditional density is constant in the sense that $\upsilon'(y|x) = r$ for some constant r, since the acceptance sets of constant size $2K$. For symmetric distributions $\Gamma(\cdot) = \Upsilon(\cdot)$ this means that $\upsilon'(y|x) = \gamma'(x|y) = r$. The first result also obtains for any other type distribution.

[13] For the uniform distribution we have

$$\Psi_1'(y) = \frac{1}{2K}\int_{y-K}^{y+K} \left(\frac{\alpha}{2}x - \frac{\alpha}{2}y\right) dx = \frac{1}{2K}\int_{-K}^{K} \left(\frac{\alpha}{2}(y+k) - \frac{\alpha}{2}y\right) dk = 0.$$

Note also that the first effect does not depend on linear densities of the type distributions, but holds in general.

Therefore, in this simple example with a multiplicative production function (which generates positive sorting) and uniform type distributions, the fixed effect of the firm is not correlated with its true type. It is independent of its true type, and the only variation might arise from small sample properties that introduce non-systematic noise. While for uniform distributions the fixed effect does not vary with the type of the firm at all, it is possible to find type distributions that generate a fixed effect that is either decreasing, increasing, or non-monotone in the type of the firm. The reason is that taking out the worker effect from the firm effect in (15) means that a high-paying firm might have a low fixed effect because it employs mainly highly paid workers, rendering this identification strategy insufficient to back out the type of the firm. Our main conclusion is that even in a model with sorting we can analytically show that the firm fixed effect is not sensitive to the firms' true type. Whether sorting is positive or negative, the fixed effect is in general ambiguous and can be zero.

4 Identifying the Strength of Sorting

We have shown that even in the simplest model it is not possible to distinguish negative from positive sorting from wage data alone, and that in a Beckerian model of the world, fixed effects estimation not only fails to identify the sign but also the strength of sorting. This raises the question whether the strength of sorting can be identified at all from wage data. In economic terms this is the more pressing issue rather than finding the sign of sorting (positive or negative) since welfare depends on matching the right types by avoiding inefficient mismatch, not on who these types are. For the purpose of the exercise we take the model literally, and see whether the model-generated data together with the assumption of a large panel allows for identification of the strength of sorting.

We will show that the identification of the strength of sorting can be obtained. Identification does not rely on the exact simplifying assumptions we made in the previous section, but obtains even when some of these assumptions are relaxed. Consider a generalization of the basic model, now with general, type-dependent search cost functions and different bargaining shares for workers and firms. Search costs might differ between workers and firms, and they vary with the type of the agent x or y. Also bargaining shares might not be equal. Denote by $c(x)$ the cost of search to the worker of type x and by $k(y)$ the cost of having job y unfilled. The worker's share of the bargaining surplus is denoted by $\gamma \in (0, 1)$.

We maintain the assumptions on $f(x, y)$ from section 3 (symmetric cross-partials in particular). To keep exposition tractable, we will also make assumptions on the search costs that ensure that matching bands are symmetric and monotone, and that higher types indeed get more when they have to wait for the second period. Symmetry implies that second-period wages are exactly the same as in the frictionless matching model. The entire analysis goes through even without symmetric matching bands, but the algebra becomes much more involved. The following assumption is sufficient to ensure this: $c'(x) = k'(x) < f_x(x, x)$ in the case of PAM, and $c'(x) = -k'(1 - x) < f_x(x, 1 - x)$ in the case of NAM. The equality ensures marginal cost symmetry and thereby the symmetry of the matching bands, the inequality ensures that second period wages minus the costs of search are increasing in type, which implies that higher worker types always get higher wages than lower worker types.[14] We also assume

[14]Differentiating the wages in (26) establishes this.

monotone matching bands which obtains if costs do not change too rapidly by type.[15] The motivation for this assumption is that it allows us to use the wages to order the firms meaningfully, despite the fact that we cannot identify which firms are the more productive ones. While we show in Section 5 that we can dispense with the symmetry assumption alltogether and how one can get identification even if matching sets are non-monotone, the current specification allows for a particularly tractable mathematical exposition.[16]

We develop a simple procedure to identify the degree of complementarity of the production technology $f(x, y)$. What we are after is to identify the magnitude of the cross-partial $|f_{xy}|$ in absolute value, which turns out to be the key determinant for the efficiency loss due to mismatch. The procedure makes use of two pieces of independent information that jointly identify the two fundamentals of the economy, the search costs $c(x), k(y)$ and the match surplus function $f(x, y)$:

1. The size of the *wage gap*: the difference between the highest and the lowest wage a given worker type receives. The wage gap allows us to identify the cost of search since the marginal wage offer must make the worker indifferent between accepting and rejecting the offer, thus relating the gross surplus (the wage gap) to the costs.[17]

2. The *matching range*: the fraction of jobs that are being accepted out of all jobs in the economy. This gives us independent information about the shape of the technology $f(x, y)$. To see this, observe that under strong complementarities, output changes fast when moving away from the Beckerian allocation. As a result, the output loss from non-assortative matching is large and only a small range of matches is accepted. In contrast, under weak or no complementarities, little or no output is lost and for a given search cost a much larger range of matches is acceptable.

In order to identify the model, we need repeated observations of wages of a given worker, and repeated observations of a given job. For some firms a job does not directly have an identifier in existing data sets, and it might be more useful to think about jobs in the same occupation within the same firm as being identical. Again, suppose for expositional purposes that we have a long panel, i.e., each worker and job goes through our two-step hiring process many times so that we observe a long set of wages for each worker and for each job. In fact, we assume that the panel is long enough that we can abstract from issues of finite samples. In this case, the average wage of a worker is his true average wage. We name a worker "x" if a fraction x of the other workers has a lower average wage and the remainder a higher average wage than him. Since higher type workers in our model indeed obtain

[15] Matching bands $A(x)$ and $B(y)$ are monotone if they form an interval whose boundaries are both either increasing or both decreasing in type. As shown in the previous section, this is trivally the case when costs are constant in type. It can be shown that for any f with $|f_{xy}|$ bounded away from zero there exists a bound $\psi > 0$ such that matching sets remain monotone whenever $c'(x) < \psi$ and $k'(y) < \psi$ for x, y.

[16] The identification method in equation (30) does not use any symmetry of either the cost or the production functions. Section 5 part 3 shows how monotonicity can be replaced by knowledge that costs are increasing in firm type, which naturally happens under discounting. Part 10 in that section discusses how an adapted version of this approach can be used even in settings that do not have any natural order of firms.

[17] We cannot adopt the same strategy for the firms, since we cannot rely on the "profit gap" for a given job if profits are hard to observe. Nevertheless, we will see that bargaining reflects variation on the firms side to a degree that allows identification of the firms' costs up to sign and a constant, which is sufficient to back out the loss from mismatch and the magnitude of the cross-partial in production.

higher wages on average, this recovers the true type of the worker. Then we can rank the jobs based on the quality of their workers: We name a job "$\hat{y}$" if a fraction $\hat{y}$ of all jobs have workers of lower quality. Monotone matching bands imply that the distribution of workers changes in the sense of first order stochastic dominance, and we can measure the worker quality for example by the average of the worker types employed in each job over time. Under PAM higher firm types match with better firms and we have $\hat{y} = y$, while under NAM higher firm types match with lower quality workers and we have $\hat{y} = 1 - y$.[18] For the remainder of this section, we will also indicate with a "hat" any function that deals with the transform, in which case it represents the original function under PAM, and represents the function evaluated at $1 - y$ under NAM. In particular, consider production function $\hat{f}(x, \hat{y})$ which has the following definition

$$\hat{f}(x, \hat{y}) = \begin{cases} f(x, \hat{y}) & \text{if} \quad \text{PAM} \\ f(x, 1 - \hat{y}) & \text{if} \quad \text{NAM} \end{cases} .$$

We know from Section (3) that we cannot identify from the wages whether PAM or NAM applies. The important point to notice is that independent of the case we are in, the cross-partial of $\hat{f}$ recovers the absolute value of the cross-partial of the original production function, i.e., we have $\hat{f}_{xy} = |f_{xy}|$ or more precisely $\hat{f}_{xy}(x, \hat{y}) = |f_{xy}(x, \mu^{-1}(\hat{y}))|$. We will see below that the absolute value of the cross-partial is indeed the relevant information needed to identify the loss from mismatch (together with the costs of waiting).

We now proceed along the two steps outlined above to back out the waiting costs and this cross-partial. It will be convenient to start with the second step (matching range), and then return to the first step (wage gap). We conclude this section with a parametric example of how to implement this strategy for our example class of production functions and discuss various measures of efficiency loss that can be obtained.

Matching Range. Assume that the costs of search have been identified from some variation in the data. Then the following allows a recovery of the strengh of the cross-partial, and therefore the importance of sorting. As before, with general search costs and bargaining share, the surplus will be positive and results in the acceptance of a match provided that the value from the current match outweighs the benefit from waiting:

$$f(x, y) - [w^\star(x) + \pi^\star(y) - c(x) - k(y)] \geq 0. \tag{21}$$

Given our symmetry assumption, second-period wages $w^\star(x)$ and profits $\pi^\star(y)$ are the same as in the frictionless case.[19] Rearranging and substituting the second-period wages according to (5) and profits

[18] We obtain the same ranking of firms if we rank them according to the best worker they match with, the worst worker they match with, or some other quantile in the distribution of workers they match with, since the distribution moves in the direction of first order stochastic dominance. Alternatively, we can rank firms by the highest or the lowest wage that they pay, both of which are increasing in type under PAM and decreasing under NAM when matching bands are monotone.

[19] Similar to the previous section, the matching set $A(y)$ includes all x such that equality (21) holds for (x, y), and $B(x)$ includes all y such that (21) holds for (x, y). Symmetry ensures that under PAM $A(x) = B(x)$, while under NAM $A(x) = \{1 - z : z \in B(1 - x)\}$. Then assortative matching in the second period implies that $\mu(x) = x$ under PAM and $\mu(x) = 1 - x$ under NAM, and wages are exactly as in the frictionless case.

according to (6) yields equivalently

$$f(x,y) - \left[\int_0^x f_x(x',\mu(x'))dx' + \int_0^y f_y(\mu^{-1}(y'),y')dy' + f(0,0)\right] \geq -[c(x)+k(y)]. \tag{22}$$

The left hand side of this expression gives exactly the loss in terms of efficiency between matching now and matching perfectly, since the term in brackets captures the social contribution that each type achieves under perfect matching. Recalling the definitions of $\hat{y}$ and $\hat{f}$ above, in the appendix we show that the loss on the left hand side (22) when worker x matches with firm $\hat{y}$ can be expressed as

$$L(x,\hat{y}) = -\int_{\hat{y}}^x \int_{y'}^x \hat{f}_{xy}(x',y')dx'dy'. \tag{23}$$

Workers only match when this loss is smaller than the costs of waiting: $L(x,\hat{y}) \geq -\left[c(x)+\hat{k}(\hat{y})\right]$. From the data, we can observe the lowest identity of firm with which worker x matches. Call this identity $\underline{y}(x)$. In a long panel, it is exactly the identity where the equality is binding. Therefore, we obtain the identifying equation:

$$L(x,\underline{y}(x)) = -\left[c(x)+\hat{k}(\underline{y}(x))\right] \tag{24}$$

or equivalently

$$-\int_{\underline{y}(x)}^x \int_{y'}^x \hat{f}_{xy}(x',y')dx'dy' = -\left[c(x)+\hat{k}(\underline{y}(x))\right]. \tag{25}$$

Since for each worker x the lowest firm $\underline{y}(x)$ can be observed, given knowledge of the costs of search this is a functional equation that identifies $\hat{f}_{xy}$ evaluated at $(x,\underline{y}(x))$, for all $x \in [0,1]$ with $\underline{y}(x) > 0$. It identifies the strength of the cross-partial because it compares the noise in the matching sets $(x - \underline{y}(x))$ to the noise in the wage data due to the search costs. If the wages vary substantially but matching sets are small, there must be a large loss in matching by slightly deviating from the optimal type, i.e., the cross-partial must be large.

The left hand side of functional equation (25) only provides information about some average level of the absolute value of the cross-partial accross the matching set of type x workers. We show at the end of this section that (25) provides enough information to back out the parameters for our familiy of example production functions with simple non-linear regression techniques. For classes of production functions with more parameters, we refer the reader to equation (30) which highlights that not only the average cross-partial for a given x worker but even variations of the cross-partial across different firm types for that same worker type can be identified. Clearly, the cross-partial can only be identified at points within the matching sets, since we do not observe matches between firms and workers outside the matching sets because both rather wait then match together.[20] Importantly, for the application of the identifying equation (25) it is crucial to obtain a notion of the search costs from the data, which we consider now.

Wage Gap. We now show how the difference in wages for a given worker can be used to identify the wage gap. Observe that this can be achieved from first-period wages alone, which is important since

[20]It is still possible to provide bounds on the cross-partial off the matching set, because it is not possible that output improves extremely fast outside the matching range since in such a case agents would like to match if they meet. While these bounds are relatively tight close to the actual matching set, they become fairly weak as one departs further from the matching set. See Eeckhout (2007) for a related exposition on such a phenomenon.

in our stylized environment the frictionless wages really represent a notion of continuation values (see also the continuous horizon extension in Section 5). Recall our notation, where every function with "hat" is the original function under PAM and is the function evaluated at $1-y$ under NAM. Observed first-period wages for the match of worker x with firm $\hat{y}$ are

$$\hat{w}(x,\hat{y}) = \gamma[\hat{f}(x,\hat{y}) - w^\star(x) - \hat{\pi}^\star(\hat{y}) + c(x) + \hat{k}(\hat{y})] + w^\star(x) - c(x), \tag{26}$$

representing the outside option $w^\star(x) - c(x)$ plus a share γ of the surplus.

Note that the lowest possible wage of worker x is given by his continuation value in the case where the surplus is zero: $\underline{w}(x) = w^\star(x) - c(x)$. Substituting this into (26) and observing that at $\hat{y} = x$ the production is exactly $\hat{f}(x,x) = w^\star(x) + \hat{\pi}^\star(x)$, we obtain

$$c(x) + \hat{k}(x) = \frac{\hat{w}(x,x) - \underline{w}(x)}{\gamma}, \tag{27}$$

where $\hat{w}(x,x)$ denotes the wage along the diagonal. This equation already entails most of the relevant economic insights, and therefore warrants some brief comments. Assume the bargaining share is known or inferred through the construction that we show next, then (27) entails all relevant information for the case where firm costs are constant. In such a case $\hat{w}(x,x)$ is simply the highest wage that the worker obtains, and therefore the right-hand side can be computed simply as the difference between the highest and the lowest wage, divided by the bargaining share. This can be directly fed into the right hand side of equation (25), which then allows identification of the cross-partial. Therefore, the combination of (25) and (27) are at the heart of the identification strategy, and capture the intuitive ideas described in the beginning of this section.

The final arguments provide technical details that establish that the variation in the data is in principle rich enough to uniquely back out the bargaining share, and to account for the fact that (27) gives firm costs that are evaluated at x while the right hand side of (25) needs the firm's cost evaluated at the lower bound $\underline{y}(x)$. Clearly, when matching sets are small the difference between x and $\underline{y}(x)$ is insubstantial.

To see that there is only a unique bargaining share consistent with the wage data, observe that given the assumption of a long enough panel we identify the type x of the worker and the type y of the firm correctly except for the ordering. Enough observations then allow a precise inference on the wage $\hat{w}(x,y)$ at the points in the matching set. This means that changes in the wages as matches change can be backed out from the observed wage data. In particular, the wage changes $\hat{w}_x(x,x)$ and $\hat{w}_y(x,x)$ when one moves away from the diagonal in the direction of better workers or firms can be inferred. From (5) and (6) we know that $\hat{w}^\star_x(x) = \hat{f}_x(x,x)$ and $\hat{\pi}^\star_y(\hat{y}) = \hat{f}_y(\hat{y},\hat{y})$, and therefore we obtain from differentiating (26) that

$$\hat{w}_x(x,x) = -(1-\gamma)c'(x) \text{ and } \hat{w}_y(x,x) = \gamma\hat{k}'(x). \tag{28}$$

Using these relationships allows us to simplify the following difference between the value of (27) for worker types x and x' :

$$\begin{aligned} \frac{\hat{w}(x,x) - \underline{w}(x) - [\hat{w}(x',x') - \underline{w}(x')]}{\gamma} &= c(x) + \hat{k}(x) - c(x') - \hat{k}(x') \\ \Leftrightarrow \frac{\hat{w}(x,x) - \underline{w}(x) - [\hat{w}(x',x') - \underline{w}(x')]}{\gamma} &= -\frac{1}{1-\gamma}\int_{x'}^{x} \hat{w}_x(\tilde{x},\tilde{x})d\tilde{x} + \frac{1}{\gamma}\int_{x'}^{x} \hat{w}_y(\tilde{y},\tilde{y})d\tilde{y}. \end{aligned} \tag{29}$$

This enables the identification of γ since all wage terms in (29) can be inferred from the data, and only a unique value of gamma solves this equation.

To be able to use (25), we need to recover $c(x) + \hat{k}(\underline{y}(x))$. While for small matching bands it holds that $\underline{y}(x) \approx x$ and we can safely use $c(x) + k(x)$ as identified by (27), adjustments for larger matching bands might be non-trivial. Building on (27) and on (28) yields the following equations that identify the right hand side of (25) exactly from the observable wages and the already identified bargaining power:

$$\begin{aligned} c(x) + \hat{k}(\underline{y}(x)) &= c(x) + \hat{k}(x) - [\hat{k}(x) - \hat{k}(\underline{y}(x))] \\ &= -\frac{\hat{w}(x,x) - \underline{w}(x)}{\gamma} - \int_{\underline{y}(x)}^{x} \hat{k}_y(y)dy \\ &= -\frac{\hat{w}(x,x) - \underline{w}(x) - \int_{\underline{y}(x)}^{x} \hat{w}_y(y,y)dy}{\gamma}. \end{aligned}$$

As mentioned before, if either the variation in firm costs is small (k' small means that $\hat{w}_y(y,y)$ is small) or if the matching bands are narrow and therefore $\underline{y}(x) \approx x$, then the third term in the numerator becomes negligible. In such cases the search costs can effectively be obtained by taking the difference between the maximum and the minimum wage that workers obtain without any further adjustment, as outlined in the discussion of (27).

The Parametric Example. To highlight the applicability of this identification strategy, we focus on the parametric family of production functions that we outlined as examples in (1) and (2), and consider the original case of constant and common search costs $c(x) = k(y) = c$ and equal bargaining weights $\gamma = 1/2$. Under our leading examples f^+ and f^- the loss due to mis-coordination in absolute terms is given by

$$|L(x,y)| = \frac{|\alpha|}{2}(x^\theta - y^\theta)^2.$$

By (25) we can identify the strength of sorting via equation

$$|\alpha|(x^\theta - \underline{y}(x)^\theta)^2 = 4c$$

or equivalently

$$x = \left(2\,(c/|\alpha|)^{1/2} - \underline{y}(x)^\theta\right)^{1/\theta}$$

as long as $\underline{y}(x) > 0$. The parameters $|\alpha|$ and θ in this functional form can be identified by the joint behavior of x and $\underline{y}(x)$. Simple non-linear regression techniques can assess these parameters. Knowing these parameters, it is easy to compute various statistics regarding the aggregate loss from mismatch. The simplest one is the dollar amount of lost output when agents match completely randomly relative to perfect matching (keeping search costs constant), which is simply:

$$\begin{aligned} \mathcal{G} &= \int_0^1 \int_0^1 |L(x,y)|dxdy \\ &= \frac{|\alpha|}{2} \int_0^1 \int_0^1 (x^\theta - y^\theta)^2 dxdy = |\alpha| \frac{\theta^2}{(2\theta+1)(\theta+1)^2}. \end{aligned}$$

Possibly more meaningful might be the loss from equilibrium play (including costs) relative to costless perfect matching, which is $\int_0^1 \int_0^1 \min\{|L(x,y)|, 2c\}dxdy$ since losses larger than c are capped by the search costs. Finally, one can ask about the output loss from completely random matching without costs relative to equilibrium play including the search costs, which is the difference between the two previous computations: $\mathcal{G} - \int_0^1 \int_0^1 \min\{|L(x,y)|, 2c\}dxdy = \int_0^1 \int_0^1 \min\{|L(x,y)| - 2c, 0\}dxdy$.

Given α and θ, we know exactly the cross-partial f_{xy} for all pairs x, y. This clearly depends on the normalization of the distribution functions (to uniform distributions) and the associated normalization of the technology f, but the dollar loss due to mismatch is invariant to the normalization. We can therefore obtain an indication of the efficiency loss of mismatch relative to the total wages for example.

5 Discussion

Our analysis of Becker's (1973) matching framework is rather specific. In this section we discuss questions of robustness and relate our results to other contributions in the literature.

1. **Alternative Identification Strategies.** First, we consider alternative strategies of identification, and relate our approach to work by Gautier and Teulings (2004, 2006). They assess the loss from mismatch in a sorting model. Their model is similar in spirit, but different along several substantial dimensions. They make assumptions on the output functions (log-supermodularity) that guarantee positive assortative matching from the outset, while we remain agnostic about the direction of sorting. Additionally, they assume that the output is sold in a final goods market, and therefore the price is endogenous, yielding a model of comparative advantage where all firms make equal profits. In contrast, our model is one of absolute advantage where more productive firms make more profits – but see the discussion on entry below. They use a second order Taylor expansion around the frictionless benchmark, which generates a local concept to back out the cross-partial. This generates in Gautier and Teulings (2006) a constant cross-partial along the entire matching range, while in our specification the cross-partial is allowed to vary. On the other hand they are careful in specifying measurement error and short panels, from which we have abstracted.

Despite the differences it is possible to investigate how the main ingredient in their approach fares in our environment. In particular, we can show in which environment there is a natural counterpart to their approach, and provide an extension for environments that go beyond their analysis. In the following we consider three identification strategies (A) – (C) that we depict in Figure 4 and discuss in turn. In all of these approaches our exposition does not rely on knowing the sign of the cross-partial, and we ask whether we can back out its absolute value regardless.

The main identification strategy in Section 4 relies on measuring the width of the matching bands. For a given search cost (which has to be backed out from the data), small matching bands is synonymous to rejecting a large fraction of firms, which only happens if it is important to match with the right firm. In Figure 4 we illustrate this approach as a measure of the distance between the matching bands, labeled (A).

An alternative approach arises from the observation that wage changes contain information about the local cross-partial. In particular, given the definition of the wages in (26), the cross-partial

$$\hat{w}_{xy}(x, \hat{y}) = \gamma \hat{f}_{xy}(x, \hat{y}). \tag{30}$$

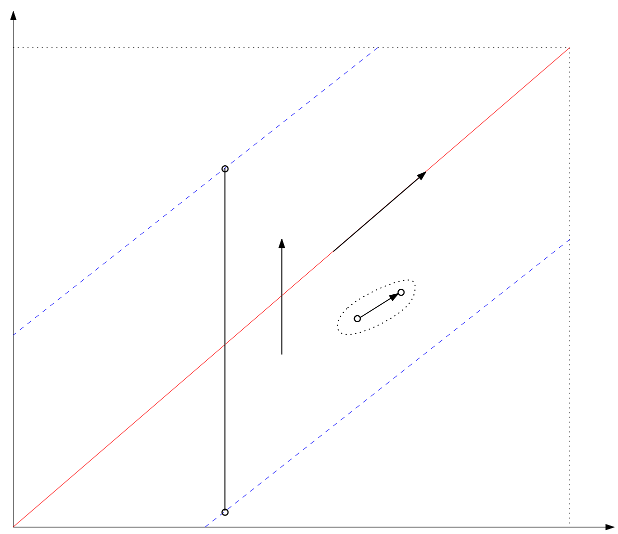

Figure 4: Identification strategies to determine the absolute value of the cross-partial: A. Global approach measuring size of and change in matching bounds; B. Local analysis of taking cross-differences of two adjacent points as in (29); C. Obtain curvature of wages along the y-axis locally around the 45-degree line.

Therefore, whenever we observe wages for $(x, \hat{y})$ and $(x', \hat{y}')$ with $x \neq x'$ and $\hat{y} \neq \hat{y}'$ and these tuples are close to each other, then the double difference in the wages $\hat{w}(x', \hat{y}') - \hat{w}(x, \hat{y}') - (\hat{w}(x', \hat{y}) - \hat{w}(x, \hat{y}))$ divided by $[x' - x][\hat{y}' - \hat{y}]$ yields an approximation for the cross-partial locally. This allows approximately the identification of the cross-partial at any point in the matching range as long as enough wage observations close by are available. This identification strategy does also not rely on any symmetry assumption regarding the costs of search or the match value function that was made in previous sections to simplify the characterization of second period wages, since the profits and wages at the second stage drop out here by taking cross-partials. We illustrate this approach in Figure 4 by the arrow labeled (B), which considers the wage change between two points (x, y) and (x', y'). Clearly, this approach only applies when the arrow in (B) is neither horizontal nor vertical, as in those cases one would divide by zero.

Gautier and Teulings (2004, 2006) exploit this idea further, and find a way to use the wage variation in the vertical direction to back out the cross-partial. In the frictionless model, worker x chooses his optimal firm $\mu(x)$ such that $w_y(x, \mu(x)) = 0$. The full derivative with respect to x along the equilibrium path gives a tight connection between the concavity of the wages and the cross-partial of the wages, and the latter is informative about the cross-partial in production according to (30). In particular, $w_{yy}(x, \mu(x)) = -w_{xy}(x, \mu(x))/\mu'(x)$. Even in the model with search frictions and without knowledge of the direction of sorting, it is easy to show that concavity of the wage in (26) yields similarly

$$\hat{w}_{yy}(x, x) = \hat{w}_{xy}(x, x) = \gamma \hat{f}_{xy}(x, x), \tag{31}$$

provided the firms' search costs are constant. That means that around the diagonal in Figure 4 we can measure the cross-partial by looking at the local curvature in wages for a given x when the firm type changes, depicted by arrow (C). This approach effectively relates to Figure 3. As search costs become small, the width of this figure becomes small and in the limit only contains firm types close to the optimal match for which (31) is approximately valid. In the limit, the relation of width to height of that figure determine the curvature $\hat{w}_{yy}$ and identifies the cross-partial (Gautier and Teulings 2004). This approach resembles approach (A), but uses the limit when search costs are small and matches only occur in the proximity of the optimal match. Alternatively, Gautier and Teulings (2006) show that the curvature of a worker's wage when the firm type changes can be directly captured in standard wage regressions by including a quadratic firm effect. This generates a ingenious way of capturing sorting in a standard econometric specification, and the coefficient directly reflects the cross-partial. Our results suggest that this holds even in the absence of restrictions on the sign of sorting.

The latter approach becomes problematic, though, when the firms' search costs are non-constant, and in particular non-linear. Undertaking the same exercise of twice differentiating (26) reveals that in general

$$\hat{w}_{yy}(x,x) = \gamma\left[\hat{f}_{xy}(x,x) + \hat{k}''(x)\right], \tag{32}$$

and therefore the curvature in a worker's wage due to changing firm type does not directly recover the cross-partial. Non-zero second derivatives of the cost function arise for example under discounting with factor β, for which the loss of the firm is $\hat{k}(y) = (1-\beta)\hat{\pi}^{\star}(\hat{y})$, which is quadratic when the output is multiplicative between the two matched types. Whether the second effect is large depends on the nature of the costs – in Gautier and Teulings (2006) this does not arise as costs are assumed to be type-independent (see their Assumption A1).

To address the possibility of type-dependent costs, one can use the adjustments we proposed in the previous section. Alternatively, we can directly exploit the cross-partial of the wages (30).

2. Infinite Horizon. The stylized nature of our two-period model was called for to be able to derive the results analytically. For estimation purposes, one would obviously choose to use an infinite horizon model as is standard in the search literature. We therefore propose the simplest model with constant search costs inspired by Atakan (2006). Time is discrete, agents are infinitely lived, and unmatched agents meet a potential partner every period. Matched partners disappear from the market until there is exogenous breakup of the match, at which point they return to the market. For expositional simplicity, consider symmetry in the distribution of types x and y and a positive cross-partial. Denote the stationary distribution of unmatched types by $G(\cdot)$. In each period, the output produced between a matched pair (x,y) is given by $f(x,y)$, where f is symmetric: $f(x,y) = f(y,x)$. If a match is rejected, the payoff is equal to $-c$. Denote by $v(x), v(y)$ the value functions of a type x and y, which are identical because of symmetry. The value function $v(x)$ for a type x is given by the wage if the job is accepted and the value of continued search in case it is rejected:

$$v(x) = \int_{\mathcal{M}(x)} w(x,y)dG(y) + \int_{y\notin\mathcal{M}(x)} dG(y)[v(x)-c] \tag{33}$$

where $w(x,y)$ is the expected wage paid over the duration of the match and $\mathcal{M}(x)$ is the matching set for type x.[21] Due to symmetry the value function is identical for firm y.[22]

Then the surplus of any match can be written as: $s(x,y) = f(x,y) - [v(x) + v(y) - 2c]$. Matches stay together as long as surplus is positive, and the lowest firm type that a worker matches with is characterized by $f(x, \underline{y}(x)) - v(x) - v(\underline{y}(x)) = -2c$. This resembles the same trade-off between current match and continuation value that we had in our two-period model. The loss is now the loss relative to optimal search, which for small costs converges to the loss in the two-period model (for the convergence result, see Atakan 2006). We assume that the surplus is split equally, so the wage is equal to the outside option $v(x)$ plus half the surplus minus the cost of search:

$$\begin{aligned} w(x,y) &= \frac{s(x,y)}{2} + v(x) - c \\ &= \frac{1}{2}\left[f(x,y) - v(x) - v(y) + 2c\right] + v(x) - c. \end{aligned} \tag{34}$$

This formulation of the wages allows us to generalize the most important points of our model: First, a fixed effects model is again misspecified because the wages at the lowest and highest firm in the matching set are equal the continuation value $v(x) - c$, and are higher in the middle, leading to a non-monotone pattern. Second, it is not hard to show that the sign of the cross-partial is still not identified, but the absolute value of the cross-partial can still be identified, for example through (30). Alternatively, the loss function between actual and optimal matching can be backed out by (24) where the loss represents the difference between output and optimal continuation, which converges to (23) for small search costs. Third, the opportunity cost of search c can also be recovered in a similar way as before, with slight adjustment: Let $\pi(x) =$Prob$\{\mathcal{M}\} = \int_{l(x)}^{u(x)} dG(y)$ be the observed probability of forming a match for worker x when unemployed, let $\mathbb{E}w(x)$ be his average wage and let $\underline{w}(x)$ be the lowest wage accepted. Since the lowest wage is equal to the continuation value: $\underline{w}(x) = v(x) - c$, we can substitute this into (33) and obtain the continuation value as the weighted average of expected and minimum wage $v(x) = \pi\mathbb{E}w(x) + (1-\pi)\underline{w}(x)$. Substituting this back into the minimum wage equation we obtain

$$c = \left[\mathbb{E}w(x) - \underline{w}\right]\pi.$$

Observe that the cost is calculated in the same way as before, except that due to random matching, now we use the average wage instead of the maximum wage, and we need to take into account the matching probability. This is quite intuitive since under random matching, the new matched wage is an average rather than the maximum and in addition, with random matching frictions next period's match is not necessarily accepted.

3. Special Cases of Type-dependent Costs: Discounting and Type-dependent Arrival Rates. Discounting is a common assumption in many models of labor search. Here we briefly outline that discounting is a special case of type-dependent search costs. Similarly, type-dependent arrival rates

[21] Note that we do not need to include the continuation value after the match breaks up exogenously because of the one-shot deviation principle, which requires optimal decisions this period and takes optimal decisions in the future as given, so that these continuation values after breakup constitute constants in the problem that can be neglected for the purpose of optimal acceptance decisions.

[22] The value function $v(y)$ for a firm is given by $v(y) = \int_{\mathcal{M}(y)}[f(x,y) - w(x,y)]dG(x) + \int_{x\notin\mathcal{M}(y)} dG(x)[v(y) - c]$. Given an equal split of the surplus and symmetry, the value functions are identical to that of the workers.

of offers also constitutes a special form of type-dependent search costs. Given this insight, it is clear that the identification strategies in Section 4 and at the beginning of this section can be applied to recover the strength of sorting. Also, we briefly outline that the problems when using the correlation of fixed effects carry over to such environments. We will start by exploring the properties of discouting because of its prominence in economic modelling, and then briefly return to type-dependent arrival rates.

In Section 2 we initially analyzed a Beckerian model with a fixed search cost, the infinite horizon version of which is analyzed by Atakan (2006). One direct result was that wages are non-monotonic in the type of the firm and, therefore, do not fulfill the assumptions of a fixed-effect estimator. In most search models, the cost of waiting is modeled by means of discounting as in Shimer and Smith (2000). We therefore now assume time discounting with factor $\beta \in (0, 1)$. In this case the costs of search are the delay costs in obtaining the second-period match: $c(x) = (1 - \beta)w^\star(x)$, $k(y) = (1 - \beta)\pi^\star(y)$, where β is the discount factor. Waiting costs are now type-dependent as the loss is proportional to the outside option: higher types pay a higher cost from delay. This is a specific case of the general setting with type-dependent costs.

When the matching sets are monotone, then our previous analysis applies and identification of the magnitude of sorting can be achieved. Under monotone matching bands we obtain an order on the firms that reflects their willingness to hire better workers. Monotone matching bands imply improvments in the distribution of matches of the firms in the sense of first order stochastic dominance and we get a consistent order, whether we order firms by the average type of the worker they employ, the minimum type or the maximum type. Shimer and Smith (2000) point out that matching bands may not be monotone under discounting, though, unless the production function is sufficiently supermodular. Otherwise matching bands may be decreasing in some range even if the match value is supermodular. Therefore, it is worthwile to point out that identification can be achieved even when matching bands are non-monotone, as long as we know that $k(y)$ is increasing as is the case under discouting.

We can then recover the order of $\hat{y}$ correctly (i.e., $\hat{y} = y$ under PAM and $\hat{y} = 1 - y$ under NAM) even when matching sets are not increasing. The reason is that under PAM the upper bound of the matching set is always increasing, while under NAM the lower bound is always decreasing. To see this, consider some firm type y and the highest worker type $x > y$ for which (22) holds. A slightly higher firm is then also willing to match with x because the right hand side of (22) decreases in firm type when costs are increasing, and under PAM the left hand side increases since the efficiency loss gets smaller as the distance between x and y shrinks. As a result, the upper bound of the matching sets are strictly increasing. Under NAM consider the lowest bound of a given y where $x < y$ and such that (22) holds. A type above y would also match with x since the right hand side of (22) has again decreased and under NAM the increase in the distance between worker x and the firm reduces the loss from mismatch. This means that the lower bound is increasing in y in the case of NAM.

Therefore, as long as we restrict ourselves to production functions that are either NAM or PAM, we can proceed as follows. First, order the firms according to the best worker they are matched with. If the matching was PAM, this recovers the firm type correctly given a long enough panel at least for those firms with interior matching sets. If the matching was NAM and matching sets are not increasing, then the non-monotonicity will imply that some firms of different type will get the same“identity” (because they have the same upper bound) and since the lower bounds of the matching sets are increasing (in $\hat{y}$) these firms have very different matching sets. When this is detected in the data, matching cannot have been PAM, and so the firms can be ordered according the lowest worker they match with, which is increasing in $\hat{y}$. Therefore, we can recover the order on the firms at least for those firms with interior matching sets, and the rest of the analysis proceeds as before. Clearly, if matching bands are monotone, this procedure does not help to distinguish PAM and NAM since ordering by either the best workers or the worst workers gives the same ranking and no inconsistencies.

While the magnitude of sorting can be recovered, we briefly highlight that similar problems arise for the use of fixed effects as we discussed in Section 3.2 for constant search costs. To show that similar caveats apply, we again focus on a simplified version of our production function $f^+(x, y) = xy$. When discounting replaces fixed waiting costs, a match between x and y will be formed provided the surplus exceeds the cost of delay: $f(x, y) - \beta x^2/2 - \beta y^2/2 > 0$. The matching set then is $A(y) = \left[\underline{K}y, \overline{K}y\right]$ where $\underline{K} = \beta^{-1}\left(1 - \sqrt{1-\beta^2}\right)$ and $\bar{K} = \beta^{-1}\left(1 + \sqrt{1-\beta^2}\right)$, and is monotone in type. The worker's wage in a match (x, y) in the first period is given by $w(x, y)$ with derivative $\partial w/\partial y$:

$$w(x, y) = \frac{1}{2}xy + \beta\frac{x^2}{4} - \beta\frac{y^2}{4} \quad \text{and} \quad \frac{\partial w(x, y)}{\partial y} = \frac{x}{2} - \beta\frac{y}{2} \tag{35}$$

The wage is hump-shaped. The derivative is negative when $x < \beta y$. Therefore, the wage is decreasing for all worker types in $[\underline{K}y, \beta y]$. This set is non-empty for any discount factor.[23] While in this setting identification of the sign of the cross-partial might be possible, the difference between wages of positive and negative assorted production functions is of order $(1 - \beta)$ and therefore hard to detect when agents are patient.[24]

Figure 5.a. illustrates the matching pattern. The matching sets are no longer constant as before. In general, they will also be non-linear (for example when output is $x^\theta y^\theta$, with $\theta \neq 1$). Figure 5.b. exhibits the wage pattern as a function of the distance k between the worker and the firm. The wages at the boundaries reflect the value from waiting and therefore necessarily are the same. The wage schedule is single-peaked, only that the peak is shifted to higher wages compared to the previous section. Still roughly for half of the acceptance set the wage is decreasing in y and falls to the value of waiting at the boundary. As we illustrated above for the case of constant search costs, going to an infinite horizon model even with discounting does not change the analysis much.

Finally, we briefly outline that discouting can be augmented by type-dependent arrival rates and still be captured as a special case of type-dependent costs. One can imagine that different types of agents have access to trading partners at different rates. Maybe more skilled workers are better connected and

[23] It is straightforward to show that $\underline{K} = \beta^{-1}\left(1 - \sqrt{1-\beta^2}\right)y < \beta y$ if and only if $1 - \beta^2 > (1 - \beta^2)^2$, which is true for all $\beta \in (0, 1)$.

[24] Under the submodular specification $f^-(x, y) = (1 - y)x + y$ the wage will be as in (35) when we replace y by its transform $\hat{y} = 1 - y$, plus an added term $\frac{1}{2}(1 - \beta)(1 - \hat{y})$. This simple example shows that identification of the sign will be difficult in praxis because the difference in wages between PAM and NAM becomes neglegible when β is close to one. In fact, one can prove that identification of the sign is impossible without restrictions on the cost functions. If the costs themselves have to be identified and there are no further restrictions on these costs, it is easy to generalize the non-identification result in Proposition 3 to the heterogeneous cost setting in Section 4. Since discounting implies the restriction that costs are increasing in type, this non-identification result does not readily apply to this setting, though.

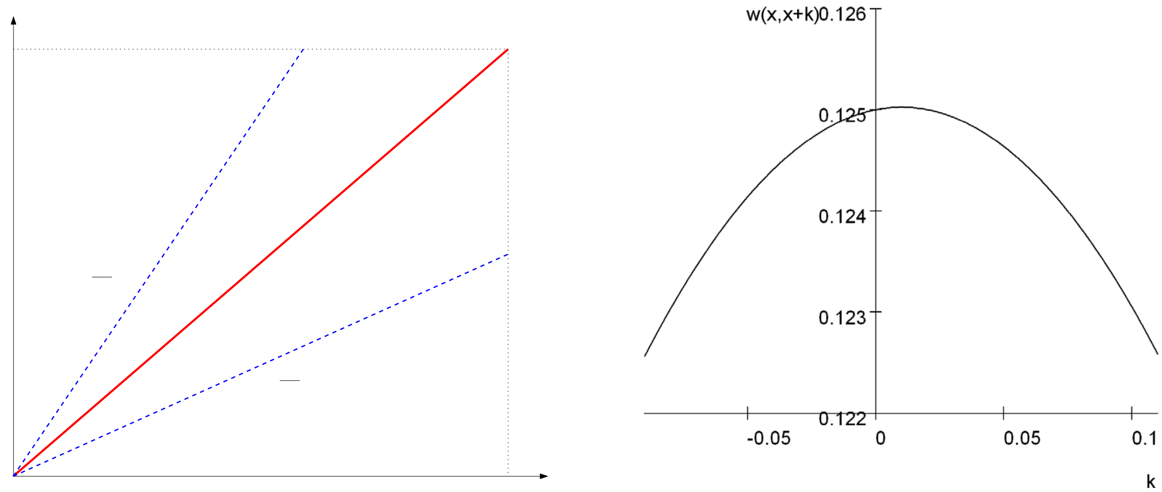

Figure 5: a. The acceptance sets under discounting. b. First period wages for under mismatch with a type that is k away from the optimal match. [Parameter values: $x = .5, \beta = .98$]

therefore more easily find a partner. As it happens, the general case of type-dependent arrival rates is isomorphic to the case of type-dependent search costs that we analyzed in Section 4. This captures the notion that different types may have more difficulty in finding a job, for example it may be harder to find a job for a high-skilled CEO than a low-skilled assembly line worker. To see that this is equivalent to the case of general search costs, observe that we can model arrival rates via a probability $\alpha(x)$ of entering the second period of our model. Then the condition is

$$f(x, y) - (\alpha(x)\beta w^\star(x) + \alpha(y)\beta\pi^\star(y)) \geq 0,$$

where as before, the equivalent cost function is $c(x) = (1 - \alpha(x)\beta)w^\star(x)$.

4. Entry. The model can easily be accommodated to allow for entry of firms at a type-dependent entry cost (see e.g., Shi (2001)). Assume that machines of quality y cost $\kappa(y)$. Clearly, as long as $\kappa(y)$ is identical to the expected profits of a firm of type y the firms are willing to enter exactly in the amounts needed to justify the distributions that we assumed in the equilibrium analysis. A sufficient condition for this to be feasible is that $f(0, 0) - w_0 > c$ and $w_0 > c$, in which case the expected payoffs for all firms and workers outweigh the costs of search and the additional element of initial entry costs leads to zero profits for all firms. Obviously, it is important that more productive machines are more expensive, otherwise all firms would acquire the same machines and the firm-type-distribution would be degenerate. While also the reverse result that any strictly increasing entry cost schedule leads to an equilibrium can be formally proven, we omit the formal exposition for brevity.[25]

[25]The formal analysis does reveal some interesting points regarding differentiability. Note that a finite number of points where f is non-differentiable do not change our analysis in the main body. With entry, it is not difficult to show that if $\kappa(y)$ and f are differentiable then the resulting type distribution will be continuous and differentiable in the frictionless case and continuous and almost everywhere differentiable in the case with search frictions. The non-differentiability may

5. Comparison with Repeated Static Matching. Consider the static wage determination model proposed by Abowd, Kramarz, Lengermann, and Perez-Duarte (2004): In each match between a pair (x, y) the total output is divided according to a sharing rule β in which the worker obtains $\beta f(x, y)$. In this specification the wage is monotonically increasing in y. When this is repeated, a high x worker who is matched with a low y job may choose, at a cost, to take a new draw. This will lead to some sorting since those who are most mismatched are most willing to take a new draw. Observe here that deviations are unilateral: a worker who is matched with a high y does not choose to separate, but it is the high y firm that chooses to take a new draw. As a result, inefficient separations occur as the worker is not permitted to compensate the separating firm for continuing the match. The worker would be willing to accept a lower wage in order to continue the match. This lower wage would eventually lead to the non-monotonicity that we see in our setting.

The important aspect is that wages are independent of outside options (assumed to be zero), which makes this model essentially static. Most labor market models do not have this feature. Rather, when deciding whether to stay together or to split up, the bargaining is usually about the surplus that the pair enjoys over and above the value that each partner can ensure himself by separating. When a firm has a high value from separating, then it first gets compensated for its high outside option and only the remaining value gets split. In such a formulation if a firm is nearly indifferent between searching for a more appropriate worker or staying with the current worker, then it is only willing to give very little additional wage to the worker (above and beyond his compensation for not searching further). In contrast, in the repeated static matching approach, such a firm would simply search for a new worker because the share it has to give to the current worker is too large (it separates even though staying matched would be socially efficient), because the wage does not reflect the continuation payoffs.

6. Directed Search. In the directed search model of Shimer (2005) with ex post screening, complementarities lead to sorting. The firms' offer strategies are monotonic, in the sense that a higher worker type obtains higher wages from better firms. Interestingly, in that model a non-monotonicity may appear on the worker side, since a higher worker type may obtain a lower wage at a given firm (being compensated by a much higher probability of getting hired). Alternatively, if workers were to auction off their labor, then the equilibrium wages offered would be again non-linear in firm type.[26]

7. On-the-job Search. On-the-Job-Search (OJS) is another likely candidate for identifying sorting using equilibrium mismatch. Bagger and Lentz (2008), Lise, Meghir and Robin (2008) and Lopes de Melo (2008) consider a sorting model with on-the-job search (as in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006)). As long as a job is scarce, matched pairs face a trade-

arise at the points where the matching sets reach the boundaries of the type space. When we re-normalize f to be able to work with a uniform type distribution, the normalized f retains monotonicity and continuity everywhere but may inherit points of non-differentiability. Since it remains almost everywhere differentiable, we can carry out our analysis as in the main body, only that the assignment function may have discontinuities in its slope at the points where normalized f is discontinuous.

[26] We are grateful to Robert Shimer for pointing this out to us. Eeckhout and Kircher (2010) show a related result of decreasing wages for better workers when directed search is introduced into the standard Becker (1973) model without ex-post screening, but in this case the result is restricted to the case of NAM.

off between matching early and waiting for the appropriate types.[27] This arises because in nearly all existing models a current match allows only the workers to search further but precludes the firms from further matches, and thus introduces an opportunity cost for inappropriate matches. Even if both sides can search such an opportunity cost remains present as long as arrival rates in a match are lower than for unmatched agents. In such cases on-the-job search will affect the exact nature of the equilibrium wages, but high type firms will still only pay relatively low wages to low type workers to compensate them for their lost continuation value without that worker. Nevertheless, the exact incorporation of on-the-job search into our environment requires further research beccause it requires a different assessment of the opportunity costs of search that go beyond the analysis presented here.

8. Use of Additional Data: The Attribution Problem. Our setup does not exploit any other data except for wages, matched partners, and possibly search duration. Existing data sets feature much richer data, both on observables of the workers and the firms. We note here the benefits and problems in using such data. First, for the workers the additional data obviously provides an additional source to identify the type of the worker. Nevertheless, when the data on wages for each worker is rich enough this is not strictly necessary, since the wages alone allow us to identifying the type of the worker. While in applications the additional data through observables reduces noise through finite samples and measurement error, for identification in long panels it is not strictly necessary.

Second, observables about firm profit or output would indeed provide additional information that goes beyond those contained in wage data. If such data on a job by job level is available, it would allow us not only to identify the strength but also the sign of sorting. And while there are good data on firm profits, the problem is that there are no data on job profits. In multi-worker firms, we need to attribute the share of each worker's contribution to the overall firm profits. Even in the simplest economy we need to decide what the contribution of very different occupations (CEO, accountant, and secretary) is to the firm profit. Since this decomposition seems difficult across occupations[28], we propose to focus on the economically important and manageable problem of identifying the strength of sorting.

Being aware of the shortcomings of the wage data in fixed effects estimates, some contributions have used output data to assess the direction of sorting, as mentioned in the introduction. For example, Mendes, van den Berg, and Lindeboom (2007) use productivity data instead of wage data. They choose *average* firm-specific productivity to attribute output from the firm to an individual worker. They find that average firm-specific productivity and worker skill exhibit strong positive sorting.

9. Measurement Error and Wage Profiles. Following most of the literature on identification, our insights are based on the assumption of large panels. In such a setting measurement error does not affect indicators such as average wages. Nevertheless, it will inflate statistics such as the maximum and deflate statistics such as the minimum wage used $\underline{w}$. Therefore, it might be more prudent to use more conservative statistics such as the 95% interval of wages for a given type of agent. The alternative

[27] In Bagger and Lentz (2008) jobs are not scarce since firms can open as many jobs as they want. The sorting effect in their model derives from differences in the intensity with which workers search for a new job.

[28] In a simple matching model of the firm, Eeckhout and Pinheiro (2008) show that only under very specific conditions, namely homotheticity in the production technology, wage ratios of different skills within a firm will be the same as those across firms. In all other cases, the wage share of a given skill is different in different firms, and as a result simply attributing profits to jobs proportional to wages will be biased.

identification procedure outlined in (30) can be applied with sufficiently many data observations if we use the average wage in a very small neighborhood of $(x, \hat{y})$ and the average wage in a very small neighborhood of $(x', \hat{y}')$ rather than each actual wage (and to the extent that the errors have mean zero).[29] In their approach to estimation Gautier and Teulings (2006) propose additional ways of incorporating measurement error and finite samples that might also be applicable to our setting. Other complications that our simplified exposition does not take care of concerns wage increases on the job, for example because of human capital accumulation, because of backloaded incentives in wage-tenure contracts, or because of outside offers that bid up the wages. As long as wages increase deterministically, the costs of search can still be backed out by the difference between the highest present value of wages in a spell of employment and the lowest present value. Issues of stochastic evolvement of wages and of short panels for workers require more careful assessment that goes beyond the current setup.

10. More General Technologies. Above, we have shown that the strength of sorting can be identified. We have explored this in a setting where higher types produce more for any given type on the other side, and where matching bands are monotone which gives some order on the firms. Yet the fact that we can achieve identification without knowledge of the sign of sorting provides a hint that the order of firms in terms of productivity is not crucial for the identification strategy. It suggests that our analysis extends to an even broader class of production functions that do not permit a clear order on firms. While we do not have a general theorem on the breath of possible production functions for which of our method delivers identification, we illustrate its applicability in a setup without any clear order on the firms. Suppose output is maximized when "similar" agents match. In such a setting there is no ranking of better jobs. Nevertheless, we can still identify how strong the complementarity is between agents with a slight adaptation our approach. Think of workers and firms as being located on a circle with circumference 1. Two types $x, y \in [0, 1]$ then have a distance $d(x, y) = \min_{k \in \{-1,0,1\}} |x - y + k|$, and the output they produce is a decreasing function of their distance given by $f(x, y) = 1 - \alpha d(x, y)^2$ for some parameter $\alpha > 0$ (this production function is studied by Gautier, Teulings and van Vuuren (2008), who additionally analyze on-the-job search). In this setup types 0 and 1 are neighbors and output of their match is high. Under the assumption of common constant search costs c, equation (27) implies that the difference between the maximum wage that a worker obtains and the minimum wage divided by his bargaining share recovers c. One way to then apply a version of equation (25) is to note that a worker who matches with a fraction z of the firms matches only with firms up to a distance $z/2$ away, and so $\underline{y}(x) = x - z/2$. Given this distance of $z/2$ between the worker type and the lowest firm type that he matches with, equation (25) immediately allows us to recover the magnitude of the cross-partial. The main point of this argument is to show that the comparison of the set of firms an agent is willing to match with against the costs from search is informative about the importance of sorting even in environments with other non-monotone production technologies.

[29] As long as the distance between $(x, \hat{y})$ and $(x', \hat{y}')$ is not completely negligible such an approach is indeed feasible using modern data sets, because of the sheer amount of available data for example in the Danish Integrated Database for Labor Market Research that essentially covers the whole population of Denmark.

6 Conclusion

In this paper we pursue two goals. First, we use the well-known model of sorting by Becker (1973) to gain insights into the wage setting process in a competitive environment, which serves as a natural benchmark. We extend that model in the smallest possible way to allow for equilibrium mismatch while retaining the basic idea underlying the assortative matching model. This allows us to provide analytical expressions for the mismatched wages in the model, to characterize the property that equilibrium wages are non-monotonic in firm type, and to provide an explicit version of a fixed effects method used in the empirical literature. We show that the latter is not well-suited to identify the sign nor the strength of sorting. Identification of the sign of sorting is in general impossible because firms pay wages based on the productivity gain from employing a higher type worker, not because they themselves are productive. The fixed effects approach is not able to identify the strength of sorting either, because even under positive sorting the wages are non-monotone in firm type. This non-monotonicity is at odds with the basic fixed-effects identifying assumption, and we show with simple examples that this precludes even a qualitative assessment of sorting because the firm fixed effect fails to correlate at all with the type of the firms.

Second, we propose to abandon the attempt to identify from wage data the sign of sorting. The mere fact that wages are determined mainly by the need of different type firms for having a better worker (which is based on the cross-partial and not the first derivative) makes such identification difficult. Under submodularity it is the low productivity firms that especially need good workers to increase their (in terms of levels) meager profits, while under supermodularity it is the productive firms that need skilled workers most. In both cases the firms that need the productive workers most have an incentive to pay high wages, which makes identification without profit (per job) data difficult. Yet in economic terms the sign of sorting may be less important than the gain that is achieved by sorting workers into the "right" job. We show that information about this gain can be identified from wage data. We propose a specific method for backing out this strength along the equilibrium path. The identification comes from determining some notion of the size of the set of firms with which a worker matches. If a worker is only willing to match with a small fraction of firms, for a given level of frictions (which can also be identified from the data) the complementarities must be large. Similarly, when a worker is willing to match with many firm types the complementarities must be weak. This gives a well-defined notion of the dollar-value of the gain from sorting in the market.

7 Appendix

Proof of Proposition 1

Proof. From equation (5) we obtain the wage schedule. When generated by an underlying production process that is supermodular f^+ this is

$$w^{\star,+}(x) = \int_0^x f_x^+(\tilde{x}, \tilde{x}) d\tilde{x} + w_0^+,$$

and for a submodular process f^- it is

$$w^{\star,-}(x) = \int_0^x f_x^-(\tilde{x}, 1-\tilde{x}) d\tilde{x} + w_0^-.$$

Observe that since $w^{\star,+}(0) = w_0^+$ and $w^{\star,-}(0) = w_0^-$, both wage schedules can be identical when the free bargaining parameter satisfies $w_0^+ = w_0^- = w_0$. Then we obtain $w^{\star,+}(x) = w^{\star,-}(x)$ for all x if $f_x^+(\tilde{x}, \tilde{x}) = f_x^-(\tilde{x}, 1-\tilde{x})$. For any $f^+(x,y)$ on $[0,1]^2$ we can define $f^-(x,y) = f^+(x, 1-y)$ on $[0,1]^2$. The only restriction is that this function may not be increasing in y, so we may need to "augment" the function to ensure that f_y is positive. If f_x, f_y are bounded, it is sufficient to add a term $\tau \cdot y$ where $\tau > 0$ is large enough to ensure $f_y > 0$ everywhere. If f_y is not bounded and negative, we need to add a function $g(y)$ that increases faster than the decrease of $f^+(x, 1-y)$ in y. ■

Fixed-Effect Decomposition in Equation (14)

Our residual is given by

$$\varepsilon_{xy} = w(x,y) - \delta(x) - \psi(y)$$

For a given firm y, the requirement that the fixed effect is unbiased requires that the average residual across the workers it matches with in the first period is zero. This means that

$$\begin{aligned} \int_{A(y)} \varepsilon_{xy} d\Gamma(x|y) &= \int_{A(y)} \left(w(x,y) - \delta(x) - \int_{A(y)} \left[w(\tilde{x}, y) - \delta(\tilde{x}) \right] d\Gamma(\tilde{x}|y) \right) d\Gamma(x|y) \qquad (36) \\ &= \int_{A(y)} \left[w(x,y) - \delta(x) \right] d\Gamma(x|y) - \int_{A(y)} \left[w(\tilde{x}, y) - \delta(\tilde{x}) \right] d\Gamma(\tilde{x}|y) = 0, \end{aligned}$$

where in the second line the double integral disappears because the interior integral is constant with respect to the argument of integration of the outer integral and the $\int_{A(y)} d\Gamma(x|y) = 1$ since $\Gamma(x|y)$ is a cumulative distribution function on $A(y)$. Similarly, for a given worker x the average residual is zero across the firms that he matches with in the first period, since

$$\begin{aligned} \int_{B(x)} \varepsilon_{xy} d\Upsilon(y|x) &= \int_{B(x)} \left(w(x,y) - \int_{B(x)} \left[w(x, \tilde{y}) - \psi(\tilde{y}) \right] d\Upsilon(\tilde{y}|x) - \psi(y) \right) d\Upsilon(y|x) \qquad (37) \\ &= \int_{B(x)} \left[w(x,y) - \psi(y) \right] d\Upsilon(y|x) - \int_{B(x)} \left[w(x, \tilde{y}) - \psi(\tilde{y}) \right] d\Upsilon(\tilde{y}|x) = 0 \end{aligned}$$

It is obvious that all transformations remain true if we use log-wages, as long as we define $w_{av}(x)$ as the average over the log-wages.

Finally, observe that our analysis is not affected by the fact that we concentrate on first period wages. Assume we look at wages across all matches. Equation (14) remains identical, but we use more data (i.e. also the second-period matches) to obtain the fixed effects. To derive the fixed effects in this case, let $\underline{a}(y) = \inf A(y), \underline{b}(x) = \inf B(x)$, $\bar{a}(y) = \sup A(y)$ and $\bar{b}(x) = \sup B(x)$ denote the boundaries of the acceptance sets for the firms and workers. Worker x and firm y matches with probability $\Upsilon(\bar{b}(x)) - \Upsilon(\underline{b}(x))$ and $\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y))$ in the first period, and with complementary probability they match in the second period.

In this case the overall fixed effects are

$$\hat{\delta}(x) = \left[\Upsilon(\bar{b}(x)) - \Upsilon(\underline{b}(x))\right] \int_{B(x)} [w(x,y) - \psi(y)]\, d\Upsilon(y|x) + \left[1 - \Upsilon(\bar{b}(x)) + \Upsilon(\underline{b}(x))\right] [w(x,\mu(x)) - \psi(\mu(x))] \quad (38)$$

$$\hat{\psi}(y) = [\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y))] \int_{A(y)} [w(x,y) - \delta(x)]\, d\Gamma(x|y) + [1 - \Gamma(\bar{a}(y)) + \Gamma(\underline{a}(y))] \left[w(\mu^{-1}(y), y) - \delta(\mu^{-1}(y))\right] \quad (39)$$

Due to our symmetry assumptions on Υ and Γ, substituting (38) into (39) yields exactly equation (17) in the main text that characterizes the firm fixed effect. Therefore, the firm fixed effect is not changed. The intuitive reason is that the firm fixed effect is governed by the *additional* wage beyond the workers average (and in second period matches this is exactly zero), and the same conclusions that we obtain for the first period wages carry over to the entire model.

Derivation of (20)

Observe that both $w(y+K,y)$ and $w(y-K,y)$ are wages for workers that are exactly indifferent between matching now and not matching. This indifference implies $w(y+K,y) = h(y+K) + \frac{\alpha(y+K)^2}{2} - c$ and $w(y-K,y) = h(y-K) + \frac{\alpha(y-K)^2}{2} - c$. Since the average wage that a worker gets when matching in the first stage is better than not matching, the difference between these wages and the average wage is negative. Using these expressions and the fact that we can write the average wage as

$$w_{av}(x) = \int_{x-K}^{x+K} w(x,y) d\Upsilon(y|x) = h(x) + \frac{\alpha}{2}x^2 - \frac{\alpha}{4}\int_{-K}^{K} k^2 \upsilon(x+k|x)dk \quad (40)$$

yields

$$\begin{aligned} \Psi_2'(y) \quad &= \quad - \left[c - \frac{\alpha}{4}\int_{-K}^{K} k^2 \upsilon(y+K+k|y+K)dk\right] \gamma(y+K|y) \\ &\quad + \left[c - \frac{\alpha}{4}\int_{-K}^{K} k^2 \upsilon(y-K+k|y-K)dk\right] \gamma(y-K|y). \end{aligned} \quad (41)$$

This effect depends on y only through the effect on the distribution. Therefore, again this effect is zero under a uniform distribution. For other distributions the effect is ambiguous because it relies on the density at the endpoints as well as on the integral over the density. In the special case where the derivative of the conditional density $\partial\upsilon(y|x)/\partial y = r$ is constant the density is linear, and so are the conditional densities. In such a case symmetry around zero of k^2 ensures that $-\frac{1}{4}\int_{-K}^{K} k^2\upsilon(Y+k|Y)dk$ is independent of Y and the terms in square brackets are identical, which directly leads to the inequalities in (20).

Derivation of (23)

First, consider the case of PAM. The loss is given by

$$
\begin{aligned}
& f(x,y) - \int_0^x f_x(x',x')dx' - \int_0^y f_y(y',y')dy' - f(0,0) \\
= \;& \int_0^y f_y(x,y')dy' + f(x,0) - \int_0^x f_x(x',x')dx' - \int_0^y f_y(y',y')dy' - f(0,0) \\
= \;& \int_0^y \int_{y'}^x f_{xy}(x',y')dx'dy' + f(x,0) - \int_0^x f_x(x',x')dx' - f(0,0) \\
= \;& \int_0^y \int_{y'}^x f_{xy}(x',y')dx'dy' - \int_0^x \int_0^{x'} f_{xy}(x',y')dy'dx' \\
= \;& \int_0^y \int_{y'}^x f_{xy}(x',y')dx'dy' - \int_0^x \int_{y'}^x f_{xy}(x',y')dx'dy' \\
= \;& -\int_y^x \int_{y'}^x f_{xy}(x',y')dx'dy'. \qquad (42)
\end{aligned}
$$

Under PAM $\hat{y} = y$ and $\hat{f}(x,y) = f(x,y)$, and so (42) yields (23).

Now consider the case of NAM. For this case, the following decomposition of the production function is useful: $f(0,1) = \int_0^1 f_y(1-y',y')dy' + f(0,0)$. It follows from the fact that the right hand side is the sum of the matched pairs $x = 0$ and $y = 1$ in the frictionless assignment under NAM, and equals exactly their joint output on the left hand side (see (3), (5) and (6)). Then the loss from mismatch can be derived as follows, where we use the equality that we just derived in line 2, line 3 follows from $\hat{f}(x,1-y) = f(x,y)$ and $\hat{y} = 1-y$, and the last line follows from similar steps that led to (42):

$$
\begin{aligned}
& f(x,y) - \int_0^x f_x(x',1-x')dx' - \int_0^y f_y(1-y',y')dy' - f(0,0). \\
= \;& f(x,y) - \int_0^x f_x(x',1-x')dx' + \int_y^1 f_y(1-y',y')dy' - f(0,1) \\
= \;& \hat{f}(x,\hat{y}) - \int_0^x \hat{f}_x(x',x')dx' - \int_{1-\hat{y}}^1 \hat{f}_y(1-y',1-y')dy' - \hat{f}(0,0) \\
= \;& \hat{f}(x,\hat{y}) - \int_0^x \hat{f}_x(x',x')dx' - \int_0^{\hat{y}} \hat{f}_y(y',y')dy' - \hat{f}(0,0) \\
= \;& -\int_{\hat{y}}^x \int_{y'}^x \hat{f}_{xy}(x',y')dx'dy'.
\end{aligned}
$$

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